Nuclear Physics and Astrophysics

PHY-302

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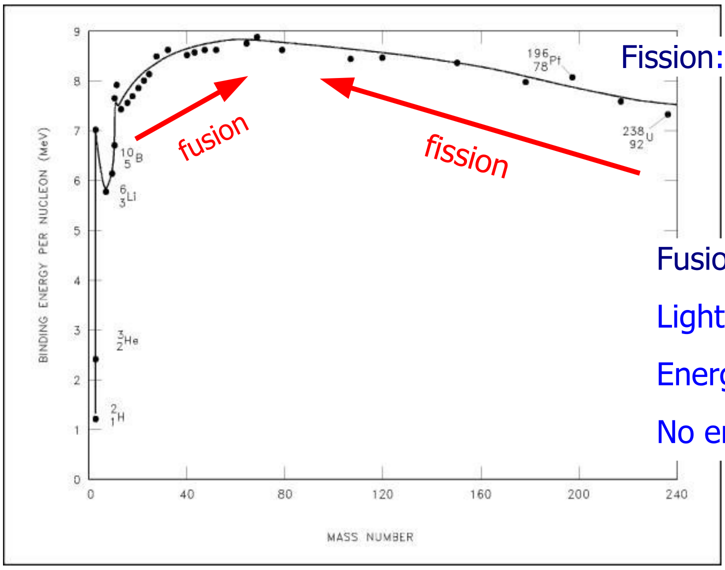
Lecture 19
Fusion

Material For This Lecture

- Nuclear Fusion
  - Theory of fusion
  - Solar fusion & the pp cycle

Fusion



Fission: gain binding energy by climbing curve

Fusion: same as fission, start with light nuclei

Light nuclei fuse creating heavier element

Energy released in process (BE increases)

No energy release for A>56 (Fe)

This process occurs in all stars - nucleosynthesis

Fusion

Energy released can be exploited: provide power

advantages over fission energy production:

- initial light nuclei are plentiful
- final products are also light stable nuclei (not heavy radioactive nuclei)

disadvantage:

- nuclei must overcome Coulomb repulsion

Consider $^{20}\text{Ne} + {}^{20}\text{Ne} \rightarrow {}^{40}\text{Ca}$

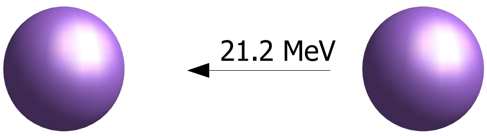
$Q = 20.7$ MeV

$= 0.5$ MeV per nucleon (comparable to fission ~200MeV/240 = 0.8)

with nuclear surfaces touching - Coulomb repulsion = 21.2 MeV

collision with 21.2 MeV initial kinetic energy yields 21.2+20.7 = 42 MeV energy

$T_f = Q + T_i$



Thus energy gain is factor of 2

question

Thermonuclear Fusion

Colliding particle beam experiments can be performed

Not viable for energy production - scattering cross section » fusion cross section

Alternative approach: heat ^{20}Ne till thermal energy overcomes Coulomb potential

each nucleus has on average (3/2) kT thermal kinetic energy:

need to overcome half Coulomb potential

Known as Thermonuclear fusion

Would require temperatures ~ 10^{11} K !!! (suns core is ~10^{7} K - cannot burn ^{20}Ne)

Simple picture yields too high critical temp for fusion...

Effective temperature for fusion is reduced by accounting for:
- QM Tunnelling
- At any temp T, some particles will have more than (3/2)kT energy
  fraction of particles at high energy for a given temp given by Maxwell-Boltzmann distribution

A basic fusion process is

$$^2H + ^2H \rightarrow ^4He + \gamma$$

Q > energy to seperate n or p
More likely process is:

$$^2H + ^2H \rightarrow ^3He + n \quad \text{Q= 3.3 MeV}$$

$$^2H + ^2H \rightarrow ^3H + p \quad \text{Q= 4.0 MeV}$$

D-D deuterium-deuterium reaction

More stable final state = more energy release

Expect ^{4}He production to have large Q - very stable

$$^2H + ^3H \rightarrow ^4He + n \quad \text{Q= 17.6 MeV}$$

D-T deuterium-tritium reaction

- Coulomb barrier in D-T is same as that of D-D reactions
- Neglecting initial kinetic energies final neutron emerges with 14.1 MeV
- Source of fast neutrons
- This reaction chosen for fusion reactors - large energy release
- But: difficult to extract energy from neutron
- Fission: neutrons carry small fraction of energy
  easier to extract energy from fission fragments

Fusion Characteristics

Initial kinetic energies for most fusion processes are small w.r.t. energy release

Temp in sun ~10^7 - 10^8 K

average kinetic energy = (3/2)kT

= (3/2) · $8.6 \times 10^{-8} \cdot 10^7$

= 1 keV

Typically thermal KE ~1-10 keV i.e. $T_{initial} \approx 0$ compared to Q ~ 17 MeV !!!

$$Q = \sum_i m_i + \sum_f m_f$$

$$= T_{final} - T_{initial}$$

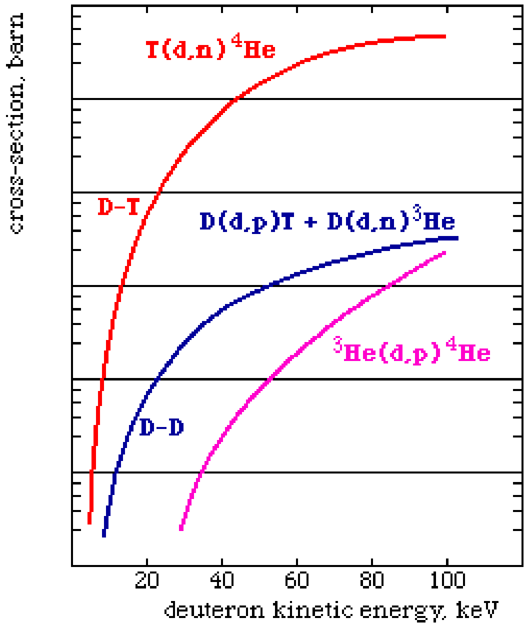
Here T means kinetic energy (not temp.) apologies for confusing notation!

Thus neglecting the initial (thermal) kinetic energy:

$$\boxed{Q_{fusion} = T_{final}}$$

The lighter particle takes the largest share of the energy (see this weeks h/w)

Simplified calulations yields this energy dependence of cross section



All fusion cross sections increase as T increases (temp, T ~ deuteron kinetic energy)

D-T reaction has highest cross section for all T

Fusion Reaction Rate

Reaction rate = σ.v for a fixed velocity v of nuclei

In thermonuclear fusion reactions particles have Maxwell-Boltzmann velocity distribution

i.e. velocity is not one fixed value. At any temp T, velocity distribution of nuclei is:

$$n(v) \propto e^{-mv^2/2kT}$$

$n(v) \cdot v^2 \cdot dv$ = relative probability to find particle in range v and v+dv for particles in thermal equilibrium at temperature T

$$\sigma \propto \frac{1}{v^2} e^{-2G}$$

Averaging over all velocities $\langle \sigma v \rangle = \int_0^\infty \frac{1}{v} \cdot e^{-2G} \cdot e^{-mv^2/2kT} v^2 dv$

or energies $= \int_0^\infty e^{-2G} \cdot e^{-E/kT} dE$

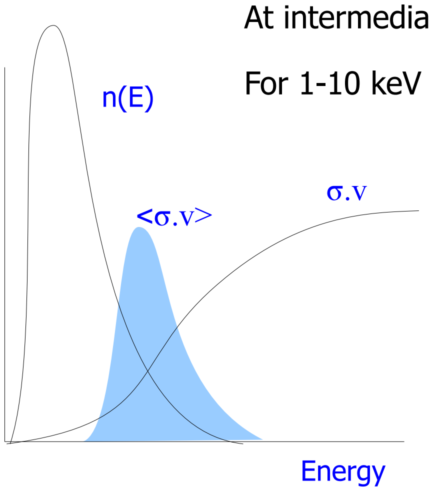
Tunnelling barrier penetration probability

At low E: Little overlap between n(E) and σ.v : <σ.v> average is low

At very high E: Maxwell distribution has small area: <σ.v> average is low

At intermediate E: <σ.v> rises to a maximum

For 1-10 keV region (T~10^7-10^8 K) D-T reaction is favoured



Critical temp for fusion is reduced:

- some particles have v. high energy in the Maxwell-Boltzmann dist.
- tunnelling of nuclei through Coulomb barrier

Note: A complete calculation would take into account that D and T are different species

Solar Fusion

Fusion process powers the sun

Occurs at very rapid rate

Constant energy output ~ 10^9 years

Basic process is fusion of hydrogen into helium

In sun: Hydrogen most abundant ~ 90%

Helium ~ 9%

other atoms < 1%

Consider only 2 particle collisions...

Solar fusion is a 3 step process, starting with proton fusion

$$^{1}H + ^{1}H \rightarrow ^{2}H + e^{+} + \nu_{e} \qquad \text{Q=1.44 MeV}$$

Note: ^{2}He production does not occur - pp bound state does not occur!

Process occurs with β^+ decay (proton converts to neutron)

Solar Fusion

This is a weak interaction - very small cross section

very low probability of occuring

occurs via exchange of heavy W boson (mass ~ 80,000 MeV/c^2)

- For solar temps (10^6-10^7 K) small cross section is partially compensated by small Coulomb repulsion
- Nevertheless reaction rate is ~ 10^{-18} s^{-1} per proton
- Solar fusion continues due to enormous number of protons ~ 10^{56}
- Due to low cross section this step in solar cycle known as 'bottleneck' slowest / least probable step in cycle

Solar Fusion Cycle

Once ^{2}H has formed then it is easy to form ^{3}He

$$^2H + ^1H \rightarrow ^3He + \gamma \qquad \text{Q=5.49 MeV}$$

question

D-D reaction is very unlikely to occur: very few produced, many more p$^+$'s around

Deuterons are therefore quickly 'eaten' to produce ^{3}He

^{3}He + ^{1}H → ^{4}Li reactions do not occur as ^{4}Li has no bound state

^{3}He + ^{2}H reactions do not occur due to lack of deuterons

Thus ^{3}He waits for the reaction:

$$^3He + ^3He \rightarrow ^4He + 2\,^1H + \gamma \qquad \text{Q=12.68 MeV}$$

Proton-proton Cycle

Three steps known as proton-proton (or pp) cycle

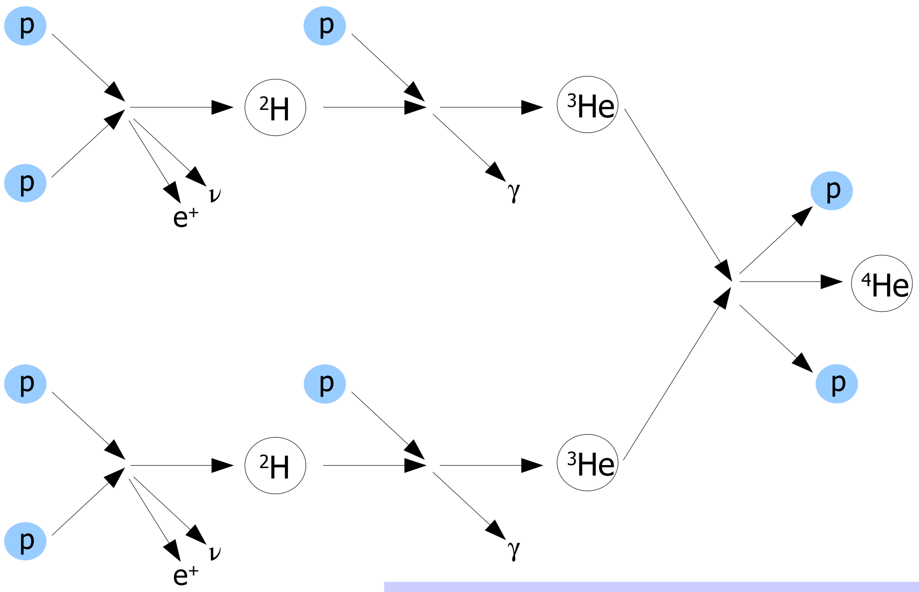
Net reaction is:

$$4\,^1H \rightarrow \ ^4He + 2e^+ + 2\nu_e + 2\gamma \qquad \text{Q=26.7 MeV}$$

Not all energy is converted to solar radiation: neutrinos escape and cause no heating of sun

- Measuring neutrino energy spectrum tells us of relative contribution of each process
- Neutrinos escape sun immediately
- Provides us with insight into reactions in core of sun!

Proton-proton Cycle



$$4\,{}^1H \rightarrow \ {}^4He + 2e^+ + 2\nu_e + 2\gamma$$ Q=26.7 MeV

Summary

Fusion process produces heavier nuclei from lighter ones
Nuclei need to overcome Coulomb barrier

- achieved at high temp through thermal kinetic energy
- QM tunnelling reduces effective temp for fusion
- high energy tail of Maxwell-Boltzmann spectrum allows drop in temp

Responsible for stellar burning
pp cycle dominates solar fusion in our sun