Chapter 9
Design of Permanent, Final Linings

Most tunnels and shafts in rock are furnished with a final lining. The common options for final lining include the following:

- Unreinforced concrete.
- Reinforced concrete.
- Segments of concrete.
- Steel backfilled with concrete or grout.
- Concrete pipe with backfill.

In many respects, tunnel and shaft lining design follows rules different from standard structural design rules. An understanding of the interaction between rock and lining material is necessary for tunnel and shaft lining design.

9-1. Selection of a Permanent Lining

The first step in lining design is to select the appropriate lining type based on the following criteria:

- Functional requirements.
- Geology and hydrology.
- Constructibility.
- Economy.

It may be necessary to select different lining systems for different lengths of the same tunnel. For example, a steel lining may be required for reaches of a pressure tunnel with low overburden or poor rock, while other reaches may require a concrete lining or no lining at all. A watertight lining may be required through permeable shatter zones or through strata with gypsum or anhydrite, but may not be required for the remainder of the tunnel. Sometimes, however, issues of constructibility will make it appropriate to select the same lining throughout. For example, a TBM tunnel going through rock of variable quality, may require a concrete segmental lining or other substantial lining in the poor areas. The remainder of the tunnel would be excavated to the same dimension, and the segmental lining might be carried through the length of the tunnel, especially if the lining is used as a reaction for TBM propulsion jacks.

a. Unlined tunnels. In the unlined tunnel, the water has direct access to the rock, and leakage will occur into or out of the tunnel. Changes in pressure can cause water to pulse in and out of a fissure, which in the long term can wash out fines and result in instability. This can also happen if the tunnel is sometimes full, sometimes empty, as for example a typical flood control tunnel. Metal ground support components can corrode, and certain rock types suffer deterioration in water, given enough time. The rough surface of an unlined tunnel results in a higher Mannings number, and a larger cross section may be required than for a lined tunnel to meet hydraulic requirements. For an unlined tunnel to be feasible, the rock must be inert to water, free of significant filled joints or faults, able to withstand the pressures in the tunnel without hydraulic jacking or other deleterious effects, and be sufficiently tight that leakage rates are acceptable. Norwegian experience indicates that typical unlined tunnels leak between 0.5 and 5 l/s/km (2.5-25 gpm/1,000 ft). Bad rock sections in an otherwise acceptable formation can be supported and sealed locally. Occasional rock falls can be expected, and rock traps to prevent debris from entering valve chambers or turbines may be required at the hydropower plant. Unlined tunnels are usually furnished with an invert pavement, consisting of 100-300 mm (4-12 in.) of unreinforced or nominally reinforced concrete, to provide a suitable surface for maintenance traffic and to decrease erosion.

b. Shotcrete lining. A shotcrete lining will provide ground support and may improve leakage and hydraulic characteristics of the tunnel. It also protects the rock against erosion and deleterious action of the water. To protect water-sensitive ground, the shotcrete should be continuous and crack-free and reinforced with wire mesh or fibers. As with unlined tunnels, shotcrete-lined tunnels are usually furnished with a cast-in-place concrete invert.

c. Unreinforced concrete lining. An unreinforced concrete lining primarily is placed to protect the rock from exposure and to provide a smooth hydraulic surface. Most shafts that are not subject to internal pressure are lined with unreinforced concrete. This type of lining is acceptable if the rock is in equilibrium prior to the concrete placement, and loads on the lining are expected to be uniform and radial. An unreinforced lining is acceptable if leakage through minor shrinkage and temperature cracks is acceptable. If the groundwater is corrosive to concrete, a tighter lining may be required to prevent corrosion by the seepage water. An unreinforced lining is generally not acceptable through soil overburden or in badly squeezing rock, which can exert nonuniform displacement loads.

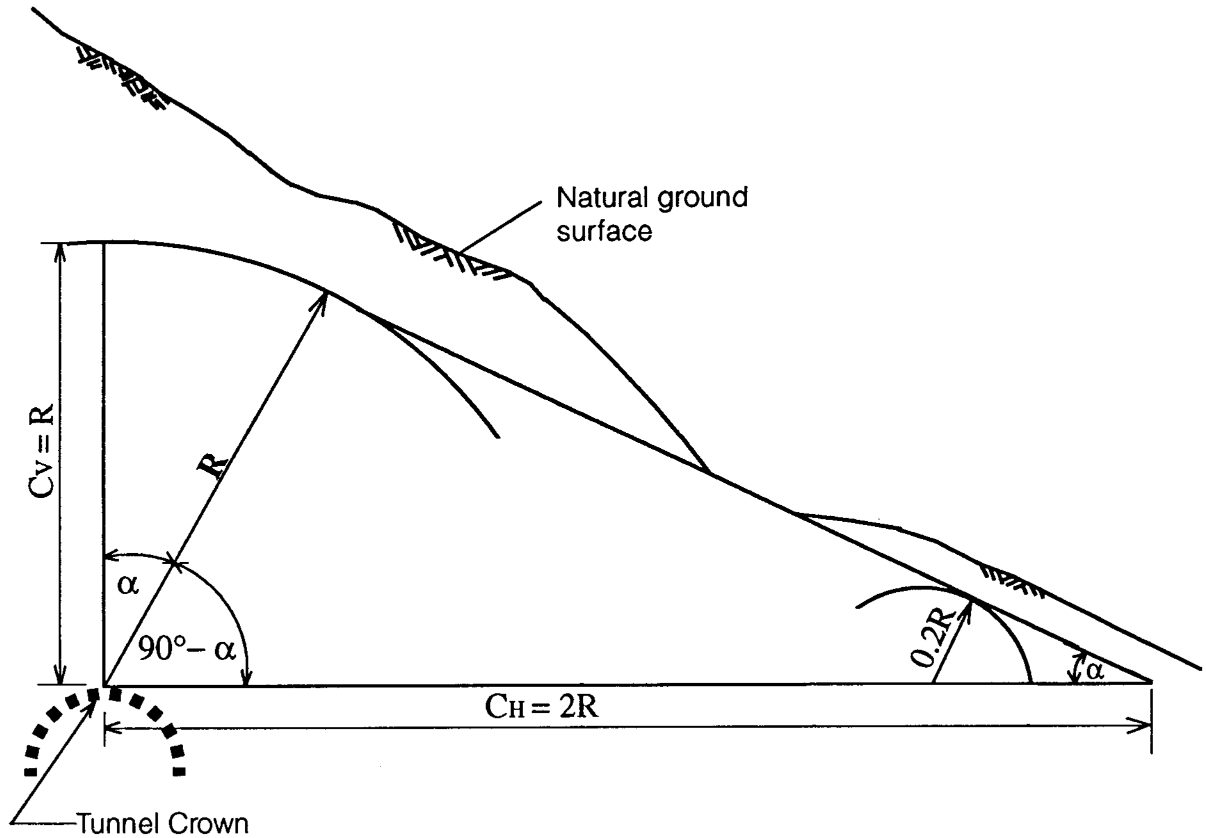
d. Reinforced concrete linings. The reinforcement layer in linings with a single layer should be placed close to the inside face of the lining to resist temperature stresses and shrinkage. This lining will remain basically undamaged for distortions up to 0.5 percent, measured as diameter change/diameter, and can remain functional for greater distortions. Multiple layers of reinforcement may be required due to large internal pressures or in a squeezing or swelling ground to resist potential nonuniform ground displacements with a minimum of distortion. It is also used where other circumstances would produce nonuniform loads, in rocks with cavities. For example, nonuniform loads also occur due to construction loads and other loads on the ground surface adjacent to shafts; hence, the upper part of a shaft lining would often require two reinforcement layers. Segmental concrete linings are often required for a tunnel excavated by a TBM. See Section 5-3 for details and selection criteria.

e. Pipe in tunnel. This method may be used for conduits of small diameter. The tunnel is driven and provided with initial ground support, and a steel or concrete pipe with smaller diameter is installed. The void around the pipe is then backfilled with lean concrete fill or, more economically, with cellular concrete. The pipe is usually concrete pipe, but steel may be required for pressure pipe. Plastic, fiber-reinforced plastic, or ceramic or clay pipes have also been used.

f. Steel lining. Where the internal tunnel pressure exceeds the external ground and groundwater pressure, a steel lining is usually required to prevent hydro-jacking of the rock. The important issue in the design of pressurized tunnels is confinement. Adequate confinement refers to the ability of a rock mass to withstand the internal pressure in an unlined tunnel. If the confinement is inadequate, hydraulic jacking may occur when hydraulic pressure within a fracture, such as a joint or bedding plane, exceeds the total normal stress acting across the fracture. As a result, the aperture of the fracture may increase significantly, yielding an increased hydraulic conductivity, and therefore increased leakage rates. General guidance concerning adequate confinement is that the weight of the rock mass measured vertically from the pressurized waterway to the surface must be greater than the internal water pressure. While this criterion is reasonable for tunneling below relatively level ground, it is not conservative for tunnels in valley walls where internal pressures can cause failure of sidewalls. Sidewall failure occurred during the development of the Snowy Mountains Projects in Australia. As can be seen from Figure 9-1, the Snowy Mountains Power Authority considered that side cover is less effective in terms of confinement as compared with vertical cover. Figure 9-2 shows guidance developed in Norway after several incidents of sidewall failure had taken place that takes into account the steepness of the adjacent valley wall. According to Electric Power Research Institute (EPRI) (1987), the Australian and the Norwegian criteria, as outlined in Figures 9-1 and 9-2, usually are compatible with actual project performance. However, they must be used with care, and irregular topographic noses and surficial deposits should not be considered in the calculation of confinement. Hydraulic jacking tests or other stress measurements should be performed to confirm the adequacy of confinement.

g. Lining leakage. It must be recognized that leakage through permeable geologic features can occur despite adequate confinement, and that leakage through discontinuities with erodible gouge can increase with time. Leakage around or through concrete linings in gypsum, porous limestone, and in discontinuity fillings containing porous or flaky calcite can lead to cavern formation and collapse. Leakage from pressured waterways can lead to surface spring formation, mudslides, and induced landslides. This can occur when the phreatic surface is increased above the original water table by filling of the tunnel, the rock mass is permeable, and/or the valleyside is covered by less permeable materials.

h. Temporary or permanent drainage. It may not be necessary or reasonable to design a lining for external water pressure. During operations, internal pressures in the tunnel are often not very different from the in situ formation water pressure, and leakage quantities are acceptable. However, during construction, inspection, and maintenance, the tunnel must be drained. External water pressure can be reduced or nearly eliminated by providing drainage through the lining. This can be accomplished by installing drain pipes into the rock or by applying filter strips around the lining exterior, leading to drain pipes. Filter strips and drains into the ground usually cannot be maintained; drain collectors in the tunnel should be designed so they can be flushed and cleaned. If groundwater inflows during construction are too large to handle, a grouting program can be instituted to reduce the flow. The lining should be designed to withstand a proportion of the total external water pressure because the drains cannot reduce the pressures to zero, and there is always a chance that some drains will clog. With proper drainage, the design water pressure may be taken as the lesser of 25 percent of the full pressure and a pressure equivalent to a column of water three tunnel diameters high. For construction conditions, a lower design pressure can be chosen.



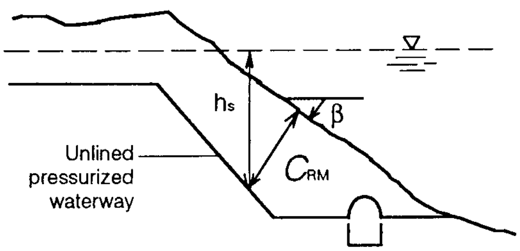
Current practice, equivalent cover. From *Unlined Tunnels of the Snowy Mountains,* Hydroelectric Authority, ASCE Conference, Oct. 1963.

Figure 9-1. Snowy Mountains criterion for confinement

9-2. General Principles of Rock-Lining Interaction

The most important material for the stability of a tunnel is the rock mass, which accepts most or all of the distress caused by the excavation of the tunnel opening by redistributing stress around the opening. The rock support and lining contribute mostly by providing a measure of confinement. A lining placed in an excavated opening that has reached stability (with or without initial rock support) will experience no stresses except due to self-weight. On the other hand, a lining placed in an excavated opening in an elastic rock mass at the time that 70 percent of all latent motion has taken place will experience stresses from the release of the remaining 30 percent of displacement. The actual stresses and displacements will depend on the modulus of the rock mass and that of the tunnel lining material. If the modulus or the in situ stress is anisotropic, the lining will distort, as the lining material deforms as the rock relaxes. As the lining material pushes against the rock, the rock load increases.

a. Failure modes for concrete linings. Conventional safety factors are the ratio between a load that causes failure or collapse of a structure and the actual or design load (capacity/load or strength/stress). The rock load on tunnel ground support depends on the interaction between the rock and the rock support, and overstress can often be alleviated by making the rock support more flexible. It is possible to redefine the safety factor for a lining by the ratio of the



C_{RM} = minimum rock cover = $h_s \gamma_W F / \gamma_R \cos\beta$;
h_s = static head; γ_W = unit weight of water;
γ_R = unit weight of rock; β = slope angle
(varies along slope); F = safety factor

Figure 9-2. Norwegian criterion for confinement

stress that would cause failure and the actual induced stress for a particular failure mechanism. Failure modes for concrete linings include collapse, excessive leakage, and accelerated corrosion. Compressive yield in reinforcing steel or concrete is also a failure mode; however, tension cracks in concrete usually do not result in unacceptable performance.

b. Cracking in tunnel or shaft lining. A circular concrete lining with a uniform external load will experience a uniform compressive stress (hoop stress). If the lining is subjected to a nonuniform load or distortion, moments will develop resulting in tensile stresses at the exterior face of the lining, compressive stresses at the interior face at some points, and tension at other points. Tension will occur if the moment is large enough to overcome the hoop compressive stress in the lining and the tensile strength of the concrete is exceeded. If the lining were free to move under the nonuniform loading, tension cracks could cause a collapse mechanism. Such a collapse mechanism, however, is not applicable to a concrete lining in rock; rock loads are typically not following loads, i.e., their intensity decreases as the lining is displaced in response to the loads; and distortion of the lining increases the loads on the lining and deformation toward the surrounding medium. These effects reduce the rock loads in highly stressed rock masses and increase them when stresses are low, thus counteracting the postulated failure mechanism when the lining has flexibility. Tension cracks may add flexibility and encourage a more uniform loading of the lining. If tension cracks do occur in a concrete lining, they are not likely to penetrate the full thickness of the lining because the lining is subjected to radial loads and the net loads are compressive. If a tension crack is created at the inside lining face, the cross-section area is reduced resulting in higher compressive stresses at the exterior, arresting the crack. Tension cracks are unlikely to create loose blocks. Calculated tension cracks at the lining exterior may be fictitious because the rock outside the concrete lining is typically in compression, and shear bond between concrete and rock will tend to prevent a tension crack in the concrete. In any event, such tension cracks have no consequence for the stability of the lining because they cannot form a failure mechanism until the lining also fails in compression. The above concepts apply to circular linings. Noncircular openings (horseshoe-shaped, for example) are less forgiving, and tension cracks must be examined for their contribution to a potential failure mode, especially when generated by following loads.

c. Following loads. Following loads are loads that persist independently of displacement. The typical example is the hydrostatic load from formation water. Fortunately the hydrostatic load is uniform and the circular shape is ideal to resist this load. Other following loads include those resulting from swelling and squeezing rock displacements, which are not usually uniform and can result in substantial distortions and bending failure of tunnel linings.

9-3. Design Cases and Load Factors for Design

The requirements of EM 1110-2-2104 shall apply to the design of concrete tunnels untless otherwise stated herein. Selected load factors for water tunnels are shown in Table 9-1. These load factors are, in some instances, different from load factors used for surface structures in order to consider the particular environment and behavior of underground structures. On occasion there may be loads other than those shown in Table 9-1, for which other design cases and load factors must be devised. Combinations of loads other than those shown may produce less favorable conditions. Design load cases and factors should be carefully evaluated for each tunnel design.

9-4. Design of Permanent Concrete Linings

Concrete linings required for tunnels, shafts, or other underground structures must be designed to meet functional criteria for water tightness, hydraulic smoothness, durability, strength, appearance, and internal loads. The lining must also be designed for interaction with the surrounding rock mass and the hydrologic regime in the rock and consider constructibility and economy.

Table 9-1
Design Cases and Recommended Load Factors for Water Tunnel[1]

Load	1	2	3	4
Dead load[2]	1.3	1.1	1.1	1.1
Rock load[3]	1.4	1.2	1.4	1.2
Hydrostatic operational[4]	1.4	-	-	-
Hydrostatic transient[5]	-	1.1	-	-
Hydrostatic external[6]	-	-	1.4	1.4
Live load				1.4

[1] This table applies to reinforced concrete linings.
[2] Self-weight of the lining, plus the weight of permanent fixtures, if any. Live load, for example, vehicles in the tunnel, would generally have a load factor of 1.4. In water tunnels, this load is usually absent during operations.
[3] Rock loads are the loads and/or distortions derived from rock-structure interaction assessments.
[4] Maximum internal pressure, minus the minimum external water pressure, under normal operating conditions.
[5] Maximum transient internal pressure, for example, due to water hammer, minus the minimum external water pressure.
[6] Maximum groundwater pressure acting on an empty tunnel.
Note: The effects of net internal hydrostatic loads on the concrete lining may be reduced or eliminated by considering interaction between lining and the surrounding rock, as discussed in Section 8-5.

a. Lining thickness and concrete cover over steel. For most tunnels and shafts, the thickness of concrete lining is determined by practical constructibility considerations rather than structural requirements. Only for deep tunnels required to accept large external hydrostatic loads, or tunnels subjected to high, nonuniform loads or distortions, will structural requirements govern the tunnel lining thickness. For concrete placed with a slick-line, the minimum practical lining thickness is about 230 mm (9 in.), but most linings, however, require a thickness of 300 mm (12 in.) or more. Concrete clear cover over steel in underground water conveyance structures is usually taken as 100 mm (4 in.) where exposed to the ground and 75 mm (3 in.) for the inside surface. These thicknesses are greater than normally used for concrete structures and allow for misalignment during concrete placement, abrasion and cavitation effects, and long-term exposure to water. Tunnels and other underground structures exposed to aggressive corrosion or abrasion conditions may require additional cover. EM 1110-2-2104 provides additional guidance concerning concrete cover.

b. Concrete mix design. EM 1110-2-2000 should be followed in the selection of concrete mix for underground works. Functional requirements for underground concrete and special constructibility requirements are outlined below. For most underground work, a 28-day compressive strength of 21 MPa (3,000 psi) and a water/cement ratio less than 0.45 is satisfactory. Higher strengths, up to about 35 MPa (5,000 psi) may be justified to achieve a thinner lining, better durability or abrasion resistance, or a higher modulus. One-pass segmental linings may require a concrete strength of 42 MPa (6,000 psi) or higher. Concrete for tunnel linings is placed during the day, cured overnight, and forms moved the next shift for the next pour. Hence, the concrete may be required to have attained sufficient strength after 12 hr to make form removal possible. The required 12-hr strength will vary depending on the actual loads on the lining at the time of form removal. Concrete must often be transported long distances through the tunnel to reach the location where it is pumped into the lining forms. The mix design must result in a pumpable concrete with a slump of 100 to 125 mm (4 to 5 in.) often up to 90 min after mixing. Accelerators may be added and mixed into the concrete just before placement in the lining forms. Functionality, durability, and workability requirements may conflict with each other in the selection of the concrete mix. Testing of trial mixes should include 12-hr strength testing to verify form removal times.

c. Reinforcing steel for crack control. The tensile strain in concrete due to curing shrinkage is of the order of 0.05 percent. Additional tensile strains can result from long-term exposure to the atmosphere (carbonization and other effects) and temperature variations. In a tunnel carrying water, these long-term effects are generally small. Unless cracking due to shrinkage is controlled, the cracks will occur at a few discrete locations, usually controlled by variations in concrete thickness, such as rock overbreak areas or at steel rib locations. The concrete lining is cast against a rough rock surface, incorporating initial ground support elements such as shotcrete, dowels, or steel sets; therefore, the concrete is interlocked with the rock in the longitudinal direction. Incorporation of expansion joints therefore has little effect on the formation and control of cracks. Concrete linings should be placed without expansion joints, and reinforcing steel should be continued across construction joints. Tunnel linings have been constructed using concrete with polypropylene olefin or steel fibers for crack control in lieu of reinforcing steel. Experience with the use of fibers for this purpose, however, is limited at the time of this writing. In tunnels, shrinkage reinforcement is usually 0.28 percent of the cross-sectional area. For highly corrosive conditions, up to 0.4 percent is used. Where large overbreaks are foreseen in a tunnel excavated by blasting, the concrete thickness should be taken as the theoretical concrete thickness plus one-half the estimated typical overbreak dimension.

d. Concrete linings for external hydrostatic load. Concrete linings placed without provisions for drainage should be designed for the full formation water pressure acting on the outside face. If the internal operating pressure is greater than the formation water pressure, the external water pressure should be taken equal to the internal operating pressure, because leakage from the tunnel may have increased the formation water pressure in the immediate vicinity of the tunnel. If the lining thickness is less than one-tenth the tunnel radius, the concrete stress can be found from the equation

$$f_c = pR/t \qquad (9\text{-}1)$$

where

f_c = stress in concrete lining

p = external water pressure

R = radius to circumferential centerline of lining

t = lining thickness

For a slender lining, out-of-roundness should be considered using the estimated radial deviation from a circular shape u_o. The estimated value of u_o should be compatible with specified roundness construction tolerances for the completed lining.

$$f_c = pR/t \pm 6pRu_o/(t^2\ (1-p/p_{cr})) \qquad (9\text{-}2)$$

where p_{cr} is the critical buckling pressure determined by Equation 9-3.

$$p_{cr} = 3EI/R^3 \qquad (9\text{-}3)$$

If the lining thickness is greater than one-tenth the tunnel radius, a more accurate equation for the maximum compressive stress at the inner surface is

$$f_c = 2pR_2^2/(R_2^2 - R_1^2) \qquad (9\text{-}4)$$

where

R_2 = radius to outer surface

R_1 = radius to inner surface of lining

e. Circular tunnels with internal pressure. Analysis and design of circular, concrete-lined rock tunnels with internal water pressure require consideration of rock-structure interaction as well as leakage control.

(1) *Rock-structure interaction.* For thin linings, rock-structure interaction for radial loads can be analyzed using simplified thin-shell equations and compatibility of radial displacements between lining and rock. Consider a lining of average radius, a, and thickness, t, subject to internal pressure, p_i, and external pressure, p_r, where Young's modulus is E_c and Poisson's Ratio is ν_c. The tangential stress in the lining is determined by Equation 9-5.

$$\sigma_t = (p_i - p_r)a/t \qquad (9\text{-}5)$$

and the relative radial displacement, assuming plane strain conditions, is shown in Equation 9-6.

$$\Delta a/a = (p_i - p_r)\ (a/t)\ ((1-v_c^2)/E_c) = (p_i - p_r)\ K_c \qquad (9\text{-}6)$$

The relative displacement of the rock interface for the internal pressure, p_r, assuming a radius of a and rock properties E_r and v_r, is determined by Equation 9-7.

$$\Delta a/a = p_r(1 + v_r)/E_r = p_r K_r \qquad (9\text{-}7)$$

Setting Equations 9-6 and 9-7 equal, the following expression for p_r is obtained:

$$p_r = p_i\ K_c/(K_c + K_r) \qquad (9\text{-}8)$$

From this is deduced the net load on the lining, p_i - p_r, the tangential stress in the lining, G_t, and the strain and/or relative radial displacement of the lining:

$$\varepsilon = \Delta\ a/a = (p_i/E_c)(a/t)(K_r/(K_r + K_c)) \qquad (9\text{-}9)$$

For thick linings, more accurate equations can be developed from thick-walled cylinder theory. However, considering the uncertainty of estimates of rock mass modulus, the increased accuracy of calculations is usually not warranted.

(2) *Estimates of lining leakage*. The crack spacing in reinforced linings can be estimated from

$$s = 5(d - 7.1) + 33.8 + 0.08\ d\rho(mm) \quad (9\text{-}10)$$

where d is the diameter of the reinforcing bars and ρ is the ratio of steel area to concrete area, A_s/A_c. For typical tunnel linings, s is approximately equal to 0.1 d/ρ. The average crack width is then $w = s\ \varepsilon$. The number of cracks in the concrete lining can then be estimated as shown in Equation 9-11.

$$n = 2\ \pi\ a/s \quad (9\text{-}11)$$

The quantity of water flow through n cracks in a lining of thickness t per unit length of tunnel can be estimated from Equation 9-12.

$$q = (n/2\eta)(\Delta p/t)\ w^3 \quad (9\text{-}12)$$

where η is the dynamic viscosity of water, and Δp is the differential water pressure across the lining. If the lining is crack-free, the leakage through the lining can be estimated from Equation 9-13.

$$q = 2\ \pi\ a\ k_c\ \Delta p/\gamma_w\ t \quad (9\text{-}13)$$

where k_c is the permeability of the concrete.

(3) *Acceptability of lining leaking*. The acceptability of leakage through cracks in the concrete lining is dependent on an evaluation of at least the following factors.

- Acceptability of loss of usable water from the system.

- Effect on hydrologic regime. Seepage into underground openings such as an underground powerhouse, or creation of springs in valley walls or lowering of groundwater tables may not be acceptable.

- Rock formations subject to erosion, dissolution, swelling, or other deleterious effects may require seepage and crack control.

- Rock stress conditions that can result in hydraulic jacking may require most or all of the hydraulic pressure to be taken by reinforcement or by an internal steel lining.

It may be necessary to assess the effects of hydraulic interaction between the rock mass and the lining. If the rock is very permeable relative to the lining, most of the driving pressure difference is lost through the lining; leakage rates can be controlled by the lining. If the rock is tight relative to the lining, then the pressure loss through the lining is small, and leakage is controlled by the rock mass. These factors can be analyzed using continuity of water flow through lining and ground, based on the equations shown above and in Chapter 3. When effects on the groundwater regime (rise in groundwater table, formation of springs, etc.) are critical, conditions can be analyzed with the help of computerized models.

f. Linings subject to bending and distortion. In most cases, the rock is stabilized at the time the concrete lining is placed, and the lining will accept loads only from water pressure (internal, external, or both). However, reinforced concrete linings may be required to be designed for circumferential bending in order to minimize cracking and avoid excessive distortions. Box 9-1 shows some general recommendations for selection of loads for design. Conditions causing circumferential bending in linings are as follows:

- Uneven support caused a thick layer of rock of much lower modulus than the surrounding rock, or a void left behind the lining.

- Uneven loading caused by a volume of rock loosened after construction, or a localized water pressure trapped in a void behind the lining.

- Displacements from uneven swelling or squeezing rock.

- Construction loads, such as from nonuniform grout pressures.

Bending reinforcement may also be required through shear zones or other zones of poor rock, even though the remainder of the tunnel may have received no reinforcement or only shrinkage reinforcement. There are many different methods available to analyze tunnel linings for bending and distortion. The most important types can be classified as follows:

Box 9-1. General Recommendations for Loads and Distortions

1. Minimum loading for bending: Vertical load uniformly distributed over the tunnel width, equal to a height of rock 0.3 times the height of the tunnel.

2. Shatter zone previously stabilized: Vertical, uniform load equal to 0.6 times the tunnel height.

3. Squeezing rock: Use pressure of 1.0 to 2.0 times tunnel height, depending on how much displacement and pressure relief is permitted before placement of concrete. Alternatively, use estimate based on elastoplastic analysis, with plastic radius no wider than one tunnel diameter.

4. For cases 1, 2, and 3, use side pressures equal to one-half the vertical pressures, or as determined from analysis with selected horizontal modulus. For excavation by explosives, increase values by 30 percent.

5. Swelling rock, saturated in situ: Use same as 3 above.

6. Swelling rock, unsaturated or with anhydrite, with free access to water: Use swell pressures estimated from swell tests.

7. Noncircular tunnel (horseshoe): Increase vertical loads by 50 percent.

8. Nonuniform grouting load, or loads due to void behind lining: Use maximum permitted grout pressure over area equal to one-quarter the tunnel diameter, maximum 1.5 m (5 ft).

- Free-standing ring subject to vertical and horizontal loads (no ground interaction).

- Continuum mechanics, closed solutions.

- Loaded ring supported by springs simulating ground interaction (many structural engineering codes).

- Continuum mechanics, numerical solutions.

The designer must select the method which best approximates the character and complexity of the conditions and the tunnel shape and size.

(1) *Continuum mechanics, closed solutions.* Moments developed in a lining are dependent on the stiffness of the lining relative to that of the rock. The relationship between relative stiffness and moment can be studied using the closed solution for elastic interaction between rock and lining. The equations for this solution are shown in Box 9-2, which also shows the basic assumptions for the solution. These assumptions are hardly ever met in real life except when a lining is installed immediately behind the advancing face of a tunnel or shaft, before elastic stresses have reached a state of plane strain equilibrium. Nonetheless, the solution is useful for examining the effects of variations in important parameters. It is noted that the maximum moment is controlled by the flexibility ratio

$$\alpha = E_r R^3/(E_c)I \quad (9\text{-}14)$$

For a large value of α (large rock mass modulus), the moment becomes very small. Conversely, for a small value (relatively rigid lining), the moment is large. If the rock mass modulus is set equal to zero, the rock does not restrain the movement of the lining, and the maximum moment is

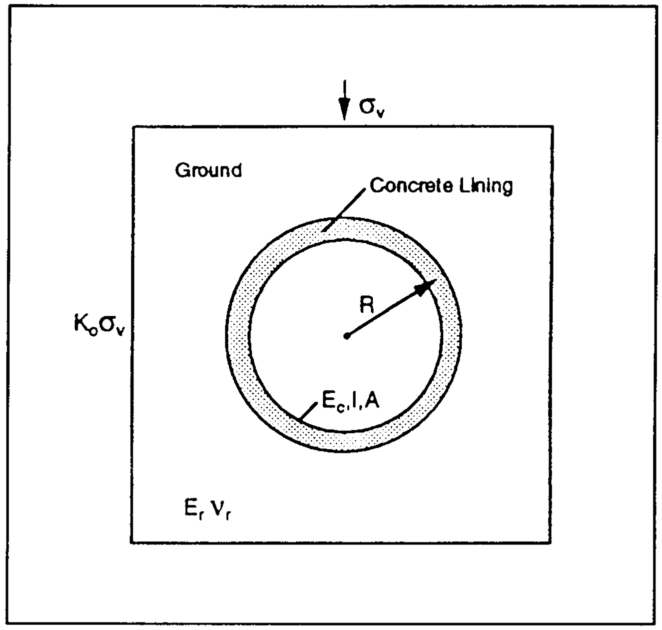
$$M = 0.25\sigma_v(1 - K_o)R^2 \quad (9\text{-}15)$$

With K_o = 1 (horizontal and vertical loads equal), the moment is zero; with K_o = 0 (corresponding to pure vertical loading of an unsupported ring), the largest moment is obtained. A few examples will show the effect of the flexibility ratio. Assume a concrete modulus of 3,600,000 psi, lining thickness 12 in. ($I = 12^3/12$), rock mass modulus 500,000 psi (modulus of a reasonably competent limestone), ν_r = 0.25, and tunnel radius of 72 in.; then α = 360, and the maximum moment

$$M = 0.0081 \times \sigma_v(1 - K_o)R^2 \quad (9\text{-}16)$$

This is a very small moment. Now consider a relatively rigid lining in a soft material: Radius 36 in., thickness 9 in., and rock mass modulus 50,000 psi (a soft shale or crushed rock); then the maximum moment is

Box 9-2. Lining in Elastic Ground, Continuum Model



Assumptions:

Plane strain, elastic radial lining pressures are equal to in situ stresses, or a proportion thereof

Includes tangetial bond between lining and ground

Lining distortion and ocmpression resisted/relieved by ground reactions

Maximum/minimum bending movement

$$M = \pm\sigma_v (1 - K_o) R^2/(4 + \frac{3 - 2v_r}{3 (1 + v_r) (1 + v_r)} \cdot \frac{E_r R^3}{E_c I}$$

Maximum/minimum hoop force

$$N = \sigma_v (1 + K_o) R/(2 + (1 - K_o) \frac{2(1 - v_r)}{(1 - 2v_r) (1 + v)} \cdot \frac{E_r R}{E_c A}) \pm \sigma_v (1 - K_o) R/(2 + \frac{4v_r E_r R^3}{(3 - 4v_r) (12(1 + v_r) E_c I + E_r R^3)}$$

Maximum/minimum radial displacement

$$\frac{u}{R} = \sigma_v (I + K_o) R^3/(\frac{2}{1 + v_r} E_r R^3 + 2E_c AR^2 + 2E_c I) \pm \sigma_v (I - K_o) R^3/(12 E_c I + \frac{3 - 2v_r}{(1 + v_r) (3 - 4v_r)} E_r R^3)$$

$$M = 0.068 \times \sigma_v(1 - K_o)R^2 \qquad (9\text{-}17)$$

It is seen that even in this instance, with a relatively rigid lining in a soft rock, the moment is reduced to about 27 percent of the moment that would be obtained in an unsupported ring. Thus, for most lining applications in rock, bending moments are expected to be small.

(2) *Analysis of moments and forces using finite elements computer programs.* Moments and forces in circular and noncircular tunnel linings can be determined using structural finite-element computer programs. Such analyses have the following advantages:

- Variable properties can be given to rock as well as lining elements.
- Irregular boundaries and shapes can be handled.
- Incremental construction loads can be analyzed, including, for example, loads from backfill grouting.
- Two-pass lining interaction can also be analyzed.

In a finite element analysis (FEM) analysis, the lining is divided into beam elements. Hinges can be introduced to simulate structural properties of the lining. Tangential and radial springs are applied at each node to simulate elastic interaction between the lining and the rock. The interface between lining and rock cannot withstand tension; therefore, interface elements may be used or the springs deactivated when tensile stresses occur. The radial and tangential spring stiffnesses, expressed in units of force/

displacement (subgrade reaction coefficient), are estimated from

$$k_r = E_r\ b\ \theta/(1 + \nu_r) \qquad (9\text{-}18)$$

$$k_t = k_r\ G/E_r = 0.5\ k_r/(1 + \nu_r) \qquad (9\text{-}19)$$

where

k_r and k_t = radial and tangential spring stiffnesses, respectively

G = shear modulus

θ = arc subtended by the beam element (radian)

b = length of tunnel element considered

If a segmental lining is considered, b can be taken as the width of the segment ring. Loads can be applied to any number of nodes, reflecting assumed vertical rock loads acting over part or all of the tunnel width, grouting loads, external loads from groundwater, asymmetric, singular rock loads, internal loads, or any other loads. Loads can be applied in stages, reflecting a sequence of construction. Figure 9-3 shows the FEM model for a two-pass lining system. The initial lining is an unbolted, segmental concrete lining, and the final lining is reinforced cast-in-place concrete with an impervious waterproofing membrane. Rigid links are used to interconnect the two linings at alternate nodes. These links transfer only axial loads and have no flexural stiffness and a minimum of axial deformation. Hinges are introduced at crown, invert, and springlines of the initial lining to represent the joints between the segments.

(3) *Continuum analysis, numerical solutions.*

Continuum analyses (Section 8-4) provide the complete stress state throughout the rock mass and the support structure. These stresses are used to calculate the (axial and shear) forces and the bending moments in the components of the support structure. The forces and moments are provided as a direct output from the computer analyses with no need for an additional calculation on the part of the user. The forces and moments give the designer information on the working load to be applied to the structure and can be used in the reinforced concrete design. Figure 9-4 shows a sample output of moment and force distribution in a lining of a circular tunnel under two different excavation conditions.

(4) *Design of concrete cross section for bending and normal force.* Once bending moment and ring thrust in a lining have been determined, or a lining distortion estimated, based on rock-structure interaction, the lining must be designed to achieve acceptable performance. Since the lining is subjected to combined normal force and bending, the analysis is conveniently carried out using the capacity-interaction curve, also called the moment-thrust diagram. EM 1110-2-2104 should be used to design reinforced concrete linings. The interaction diagram displays the envelope of acceptable combinations of bending moment and axial force in a reinforced or unreinforced concrete member. As shown in Figure 9-5, the allowable moment for low values of thrust increases with the thrust because it reduces the limiting tension across the member section. The maximum allowable moment is reached at the so-called balance point. For higher thrust, compressive stresses reduce the allowable moment. General equations to calculate points of the interaction diagram are shown in EM 1110-2-2104. Each combination of cross-section area and reinforcement results in a unique interaction diagram, and families of curves can be generated for different levels of reinforcement for a given cross section. The equations are easily set up on a computer spreadsheet, or standard structural computer codes can be used. A lining cross section is deemed adequate if the combination of moment and thrust values are within the envelope defined by the interaction diagram. The equations shown in EM 1110-2-2104 are applicable to a tunnel lining of uniform cross section with reinforcement at both interior and exterior faces. Linings with nonuniform cross sections, such as coffered segmental linings, are analyzed using slightly more complex equations, such as those shown in standard structural engineering handbooks, but based on the same principles. Tunnel lining distortion stated as a relative diameter change ($\Delta D/D$) may be derived from computerized rock-structure analyses, from estimates of long-term swelling effects, or may be a nominal distortion derived from past experience. The effect of an assumed distortion can be analyzed using the interaction diagram by converting the distortion to an equivalent bending moment in the lining. For a uniform ring structure, the conversion formula is

$$M = (3EI/R)(\Delta D/D) \qquad (9\text{-}20)$$

In the event that the lining is not properly described as a uniform ring structure, the representation of ring stiffness in this equation ($3EI/R$) should be modified. For example, joints in a segmental lining introduce a reduction in the moment of inertia of the ring that can be approximated by the equation

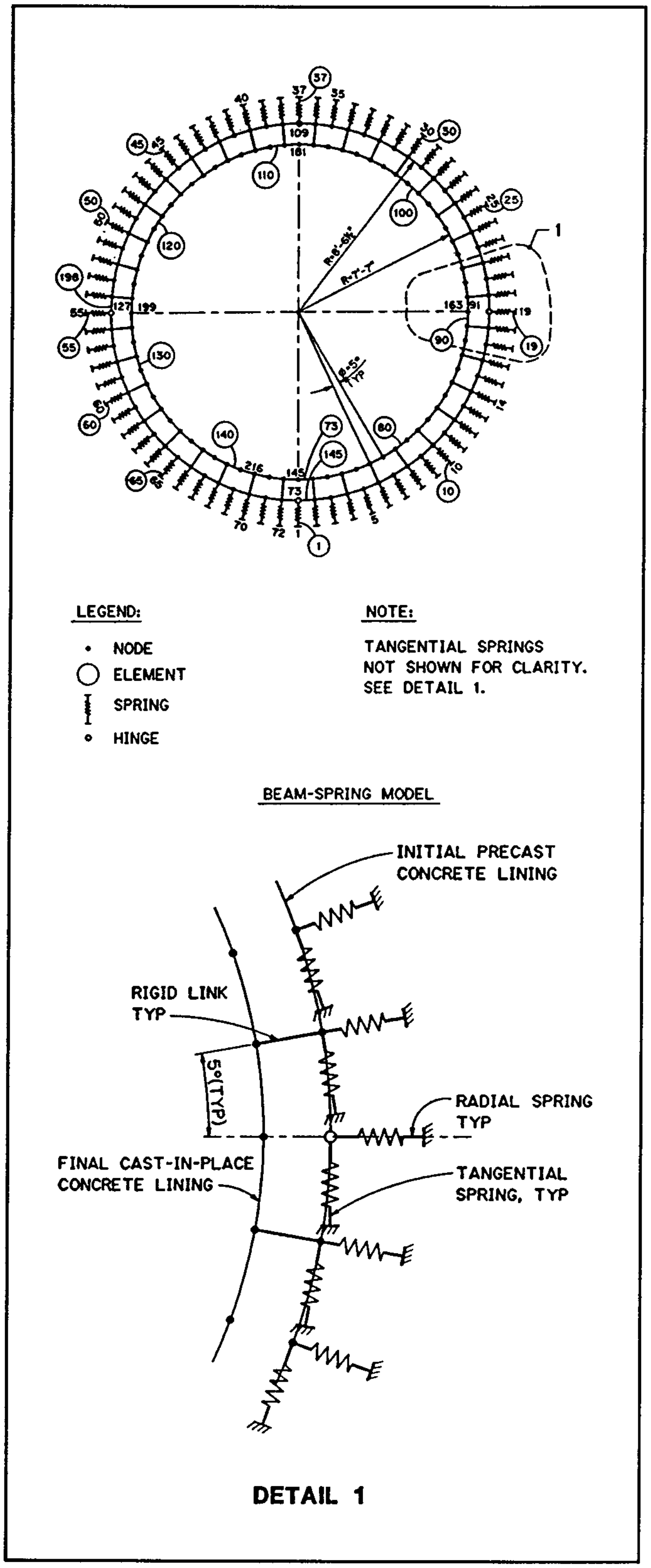


Figure 9-3. Descretization of a two-pass lining system for analysis

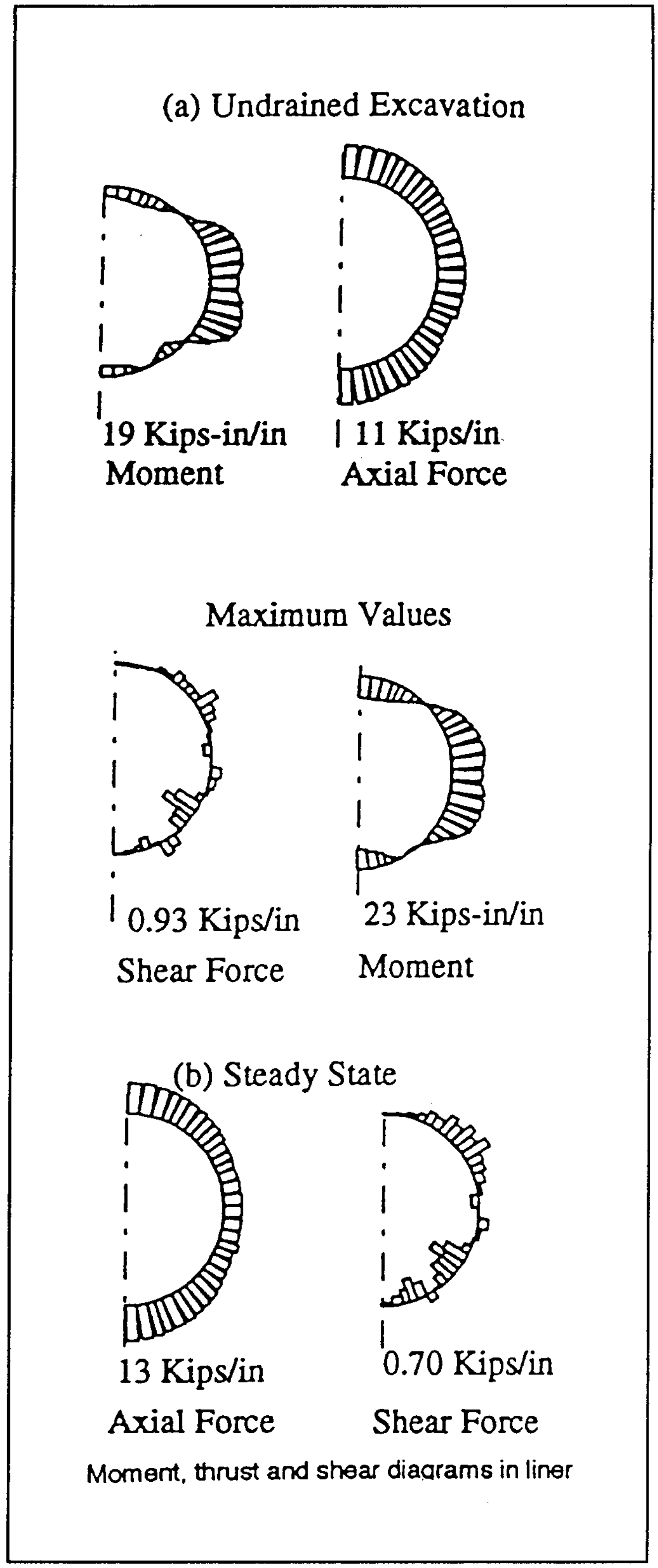


Figure 9-4. Moments and forces in lining shown in Figure 9-3

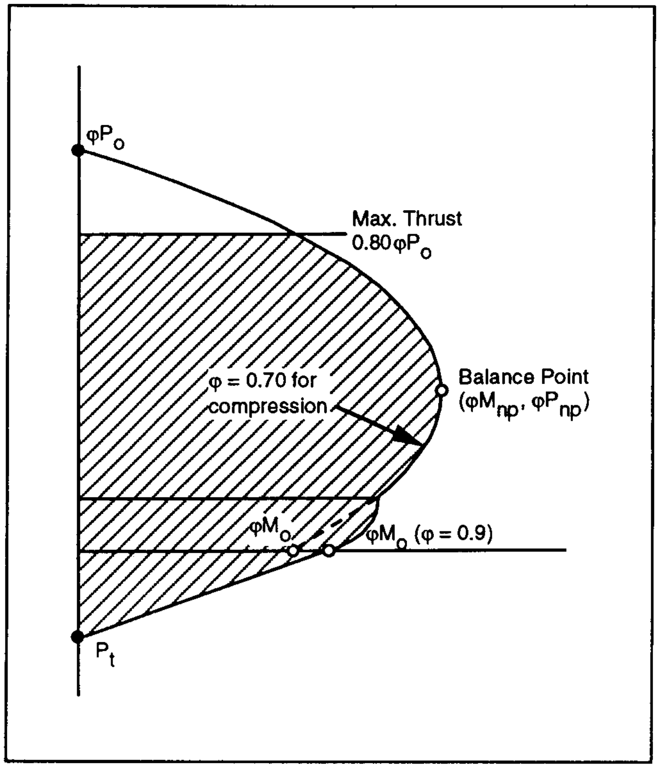


Figure 9-5. Capacity interaction curve

$$I_{eff} = I_j + (4/n)^2 I \qquad (9\text{-}21)$$

where

I = moment of inertia of the lining

I_j = moment of inertia of the joint

n = number of joints in the lining ring where $n > 4$

Alternatively, more rigorous analyses can be performed to determine the effects of joints in the lining. Nonbolted joints would have a greater effect than joints with tensioned bolts. If the estimated lining moment falls outside the envelope of the interaction diagram, the designer may choose to increase the strength of the lining. This may not always be the best option. Increasing the strength of the lining also will increase its rigidity, resulting in a greater moment transferred to the lining. It may be more effective to reduce the rigidity of the lining and thereby the moment in the lining. This can be accomplished by (a) introducing joints or increasing the number of joints and (b) using a thinner concrete section of higher strength and introducing stress relievers or yield hinges at several locations around the ring, where high moments would occur.

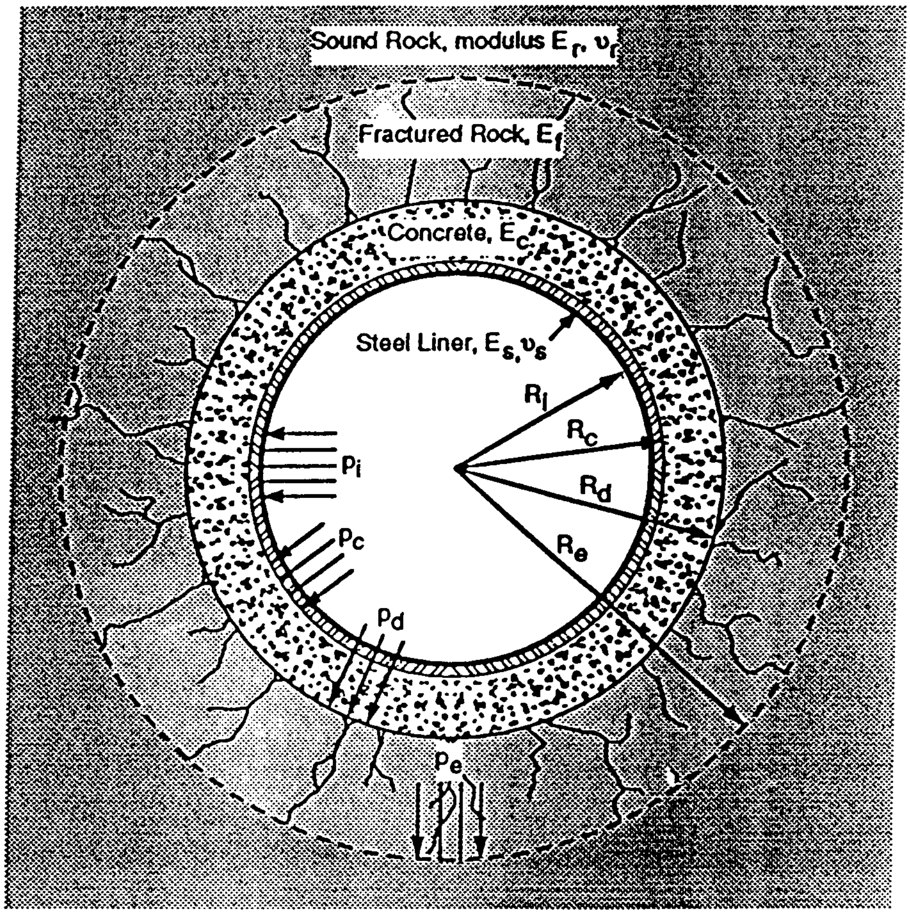
9-5. Design of Permanent Steel Linings

As discussed in Section 9-4, a steel lining is required for pressure tunnels when leakage through cracks in concrete can result in hydrofracturing of the rock or deleterious leakage. Steel linings must be designed for internal as well as for external loads where buckling is critical. When the external load is large, it is often necessary to use external stiffeners. The principles of penstock design apply, and EM 1110-2-3001 provides guidance for the design of steel penstocks. Issues of particular interest for tunnels lined with steel are discussed herein.

a. Design of steel linings for internal pressure. In soft rock, the steel lining should be designed for the net internal pressure, maximum internal pressure minus minimum external formation water pressure. When the rock mass has strength and is confined, the concrete and the rock around the steel pipe can be assumed to participate in carrying the internal pressure. Box 9-3 shows a method of analyzing the interaction between a steel liner, concrete, and a fractured or damaged rock zone, and a sound rock considering the gap between the steel and concrete caused by temperature effects. The extent of the fractured rock zone can vary from little or nothing for a TBM-excavated tunnel to one or more meters in a tunnel excavated by blasting, and the quality of the rock is not well known in advance. Therefore, the steel lining, which must be designed and manufactured before the tunnel is excavated, must be based on conservative design assumption. If the steel pipe is equipped with external stiffeners, the section area of the stiffeners should be included in the analysis for internal pressure.

b. Design considerations for external pressure. Failure of a steel liner due to external water pressure occurs by buckling, which, in most cases, manifests itself by formation of a single lobe parallel to the axis of the tunnel. Buckling occurs at a critical circumferential/ axial stress at which the steel liner becomes unstable and fails in the same way as a slender column. The failure starts at a critical pressure, which depends not only on the thickness of the steel liner but also on the gap between the steel liner and concrete backfill. Realistically, the gap can vary from 0 to 0.001 times the tunnel radius depending on a number of factors, including the effectiveness of contact grouting of voids behind the steel liner. Other factors include the effects of heat of hydration of cement, temperature changes of steel and concrete during construction, and ambient temperature changes due to forced or natural ventilation of the tunnel. For example, the steel liner may reach temperatures 80 °F or more due to ambient air temperature

Box 9-3. Interaction Between Steel Liner, Concrete and Rock



1. Assume concrete and fractured rock ar cracked; then

   $p_c R_c = p_d R_d = p_e R_e$;
   $p_d = p_c R_c / R_d$; $p_c = p_e R_c / R_e$

2. Steel lining carries pressure p_i - p_c and sustains radial displacement

   $\Delta s = (p_i - p_c)\ R_i^2\ (1 - v_s^2)\ /\ (tE_s)$

3. $\Delta s = \Delta k + \Delta c + \Delta d + \Delta E$, where
   Δk = radial temperature gap = $C_s \Delta T R_i$ ($C_s = 6.5.10^{-6}$/°F)
   Δc = compression of concrete = $(p_c R_c / E_c)\ \ln\ (R_d / R_c)$
   Δd = compression of fractured rock = $(p_c R_c / E_f)\ \ln\ (R_e / R_d)$
   Δe = compression of intact rock = $(p_c R_c / E_r)\ (1 + \upsilon_r)$

4. Hence

$$p_c = (p_i R_i^2\ (1 - \upsilon_s^2\ )\ /\ tE_s - C_s \Delta T R_c)/(R_i^2\ (1 - \upsilon_s^2)\ /\ tE_s + (R_c / E_c)\ \ln\ (R_d / R_c) + (R_c / E_f)\ \ln\ (R_e / R_d) + R_c\ (1 + \upsilon_r)\ /\ E_r)$$

and the heat of hydration. If the tunnel is dewatered during winter when the water temperature is 34 °F, the resulting difference in temperature would be 46 °F. This temperature difference would produce a gap between the steel liner and concrete backfill equal to 0.0003 times the tunnel radius. Definition of radial gap for the purpose of design should be based on the effects of temperature changes and shrinkage, not on imperfections resulting from inadequate construction. Construction problems must be remedied before the tunnel is put in operation. Stability of the steel liner depends also on the effect of its out-of-roundness. There are practical limitations on shop fabrication and field erection in controlling the out-of-roundness of a steel liner. Large-diameter liners can be fabricated with tolerance of about 0.5 percent of the diameter. In other words, permissible tolerances during fabrication and erection of a liner may permit a 1-percent difference between measured maximum and minimum diameters of its deformed (elliptical) shape. Such flattening of a liner, however, should not be considered in defining the gap used in design formulas. It is common practice, however, to specify internal spider bracing for large-diameter liners, which is adjustable to obtain the required circularity before and during placement of concrete backfill. Spider bracing may also provide support to the liner during contact grouting between the liner and concrete backfill. A steel liner must be designed to resist maximum external water pressure when the tunnel is dewatered for inspection and maintenance. The external water pressure on the steel liner can develop from a variety of sources and may be higher than the vertical distance to the ground surface due to perched aquifers. Even a small amount of water accumulated on the outside of the steel liner can result in buckling when the tunnel is dewatered for inspection or maintenance. Therefore, pressure readings should be taken prior to dewatering when significant groundwater pressure is expected. Design of thick steel liners for large diameter tunnels is subject to practical and economic limitations. Nominal thickness liners, however, have been used in large-diameter tunnels with the addition of an external drainage system consisting of steel collector pipes with drains embedded in concrete backfill. The drains are short, small-diameter pipes connecting the radial gap between the steel liner and concrete with the collectors. The collectors run parallel to the axis of the tunnel and discharge into a sump inside the power house. Control valves should be provided at the end of the collectors and closed during tunnel operations to prevent unnecessary, continuous drainage and to preclude potential clogging of the drains. The valves should be opened before dewatering of the tunnel for scheduled maintenance and inspection to allow drainage.

c. Design of steel liners without stiffeners. Analytical methods have been developed by Amstutz (1970), Jacobsen (1974), and Vaughan (1956) for determination of critical buckling pressures for cylindrical steel liners without stiffeners. Computer solutions by Moore (1960) and by MathCad have also been developed. The designer must be aware that the different theoretical solutions produce different results. It is therefore prudent to perform more than one type of analyses to determine safe critical and allowable buckling pressures. Following are discussions of the various analytical methods.

(1) *Amstutz's analysis.* Steel liner buckling begins when the external water pressure reaches a critical value. Due to low resistance to bending, the steel liner is flattened and separates from the surrounding concrete. The failure involves formation of a single lobe parallel to the axis of the tunnel. The shape of lobe due to deformation and elastic shortening of the steel liner wall is shown in Figure 9-6.

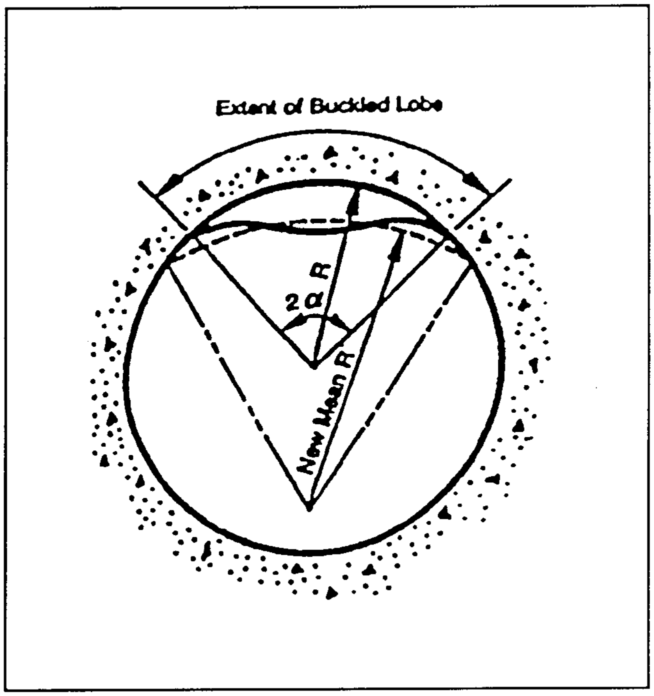


Figure 9-6. Buckling, single lobe

The equations for determining the circumferential stress in the steel-liner wall and corresponding critical external pressure are:

$$\frac{\sigma_N - \sigma_v}{\sigma_F^* - \sigma_N}\left[\left(\frac{r}{i}\right)\frac{\sqrt{\sigma_N}}{E^*}\right]^3 \cong 1.73\left(\frac{r}{e}\right)\left[1 - 0.225\left(\frac{r}{e}\right)\frac{\sigma_F^* - \sigma_N}{E^*}\right] \qquad (9\text{-}22)$$

$$P_{cr} \cong \left(\frac{F}{r}\right)\sigma_N\left(1 - 0.175\left(\frac{r}{e}\right)\frac{\sigma_F^* - \sigma_N}{E^*}\right) \qquad (9\text{-}23)$$

where

$i = t/\sqrt{12}$, $e = t/2$, $F = t$

$\sigma_v = -(k/r)E^*$

k/r = gap ratio between steel and concrete = γ

r = tunnel liner radius

t = plate thickness

E = modulus of elasticity

$E^* = E/(1-\nu^2)$

σ_y = yield strength

σ_N = circumferential/axial stress in plate liner

$\mu = 1.5-0.5[1/(1+0.002\ E/\sigma_y)]^2$

$\sigma_F^* = \mu\sigma_y\sqrt{1-\nu+\nu^2}$

ν = Poisson's Ratio

In general, buckling of a liner begins at a circumferential/axial stress (σ_N) substantially lower than the yield stress of the material except in liners with very small gap ratios and in very thick linings. In such cases σ_N approaches the yield stress. The modulus of elasticity (E) is assumed constant in Amstutz's analysis. To simplify the analysis and to reduce the number of unknown variables, Amstutz introduced a number of coefficients that remain constant and do not affect the results of calculations. These coefficients are dependent on the value of ε, an expression for the inward deformation of the liner at any point, see Figure 9-7. Amstutz indicates that the acceptable range for values of ε is $5<\varepsilon<20$. Others contend that the ε dependent coefficients are more acceptable in the range $10<\varepsilon<20$, as depicted by the flatter portions of the curves shown in Figure 9-7. According to Amstutz, axial stress (σ_N) must be determined in conjunction with the corresponding value of ε. Thus, obtained results may be considered satisfactory providing $\sigma_N<0.8\sigma_y$. Figure 9-8 shows curves based on Amstutz equations (after Moore 1960). Box 9-4 is a MathCad application of Amstutz's equations.

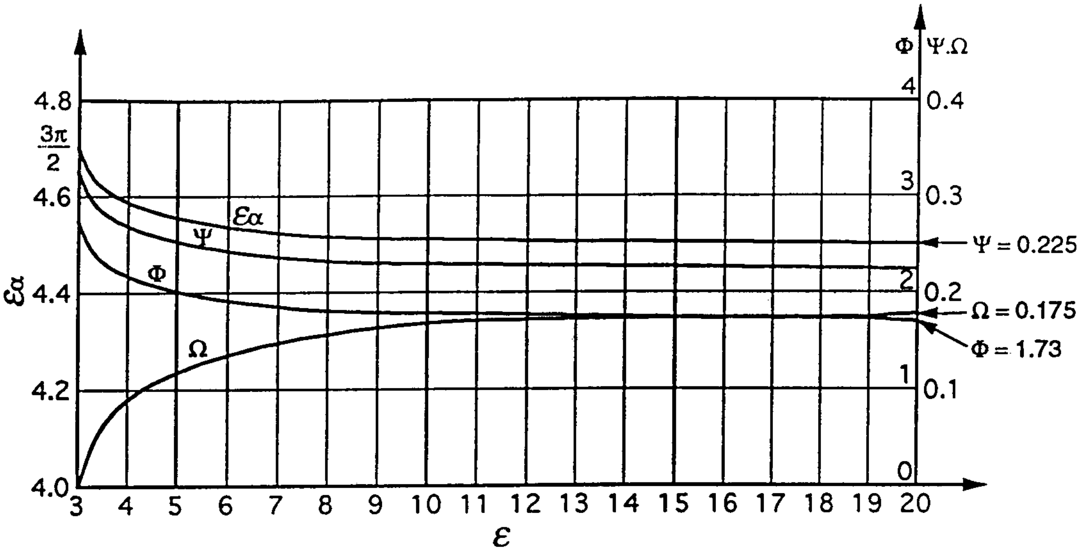
(2) *Jacobsen's analysis.* Determination of the critical external buckling pressure for cylindrical steel liners without stiffeners using Jacobsen's method requires solution of three simultaneous nonlinear equations with three unknowns. It is, however, a preferred method of design since, in most cases, it produces lower critical allowable buckling pressures than Amstutz's method. A solution of Jacobsen equations using MathCad is shown in Box 9-5.

The three equations with three unknowns α, β, and p in Jacobsen's analysis are:

$$r/t = \frac{\sqrt{[(9\pi^2/4\ \beta^2) - 1]\ [\pi - \alpha + \beta\ (\sin\alpha\ /\ \sin\beta)^2]}}{12\ (\sin\alpha/\sin\beta)^3\ [\alpha - (\pi\Delta/r) - \beta(\sin\alpha/\sin\beta)\ [1 + \tan^2(\alpha - \beta)/4]]} \qquad (9\text{-}24)$$

$$p/E^* = \frac{(9/4)\ (\pi/\beta)^2 - 1}{12\ (r/t)^3\ (\sin\alpha/\sin\beta)^3} \qquad (9\text{-}25)$$

$$\sigma_y/E^* = (t/2r)\ [1 - (\sin\beta/\sin\alpha)] + (pr\sin\alpha/E^*\ t\sin\beta)\left[1 + \frac{4\beta\ r\sin\alpha\tan(\alpha-\beta)}{\pi\ t\sin\beta}\right] \qquad (9\text{-}26)$$



Note: At $\epsilon = 2$, $\alpha = 180°$ ($\epsilon\alpha = 360°$) and Φ and $\Psi \rightarrow \infty$

ϵ	$\epsilon\alpha°$	$\alpha°$	$\tan \epsilon\alpha$	$\tan \alpha$	$\epsilon \tan \alpha$	$\cos(\epsilon\alpha)$	$\sin(\epsilon\alpha)$	$\sin \alpha$	$\epsilon\alpha$	β (25)	γ (29)	δ (35)	Φ (39)	Ψ (40)	Ω (48)
3	270°00'	90°00'	∞	∞	∞	0	-1.00000	-1.0000	4.71239	-2.6667	28.3	8.00	2.88	0.331	0
4	263°37',2	65°54',3	8.9446	2.2360	8.9440	-0.11112	-0.99381	0.91287	4.60104	-1.8095	32.7	16.67	2.21	0.271	0.100
5	261°11',6	52°14',3	6.4550	1.2910	6.4550	-0.15310	-0.98821	0.79056	4.55868	-1.3933	38.7	27.67	2.00	0.251	0.133
10	258°19',7	25°50'	4.8409	0.48413	4.8413	-0.20231	-0.97932	0.43575	4.50868	-0.6650	71.4	119.03	1.78	0.226	0.168
20	257°40',2	12°53'	4.5749	0.22873	4.5746	-0.21357	-0.97693	0.22297	4.49719	-0.3286	143.4	484.2	1.73	0.225	0.175

Figure 9-7. Amstutz coefficients as functions of "ε"

where

α = one-half the angle subtended to the center of the cylindrical shell by the buckled lobe

β = one-half the angle subtended by the new mean radius through the half waves of the buckled lobe

p = critical external buckling pressure, psi

Δ/r = gap ratio, for gap between steel and concrete

r = tunnel liner internal radius, in.

σ_y = yield stress of liner, psi

t = liner plate thickness, in.

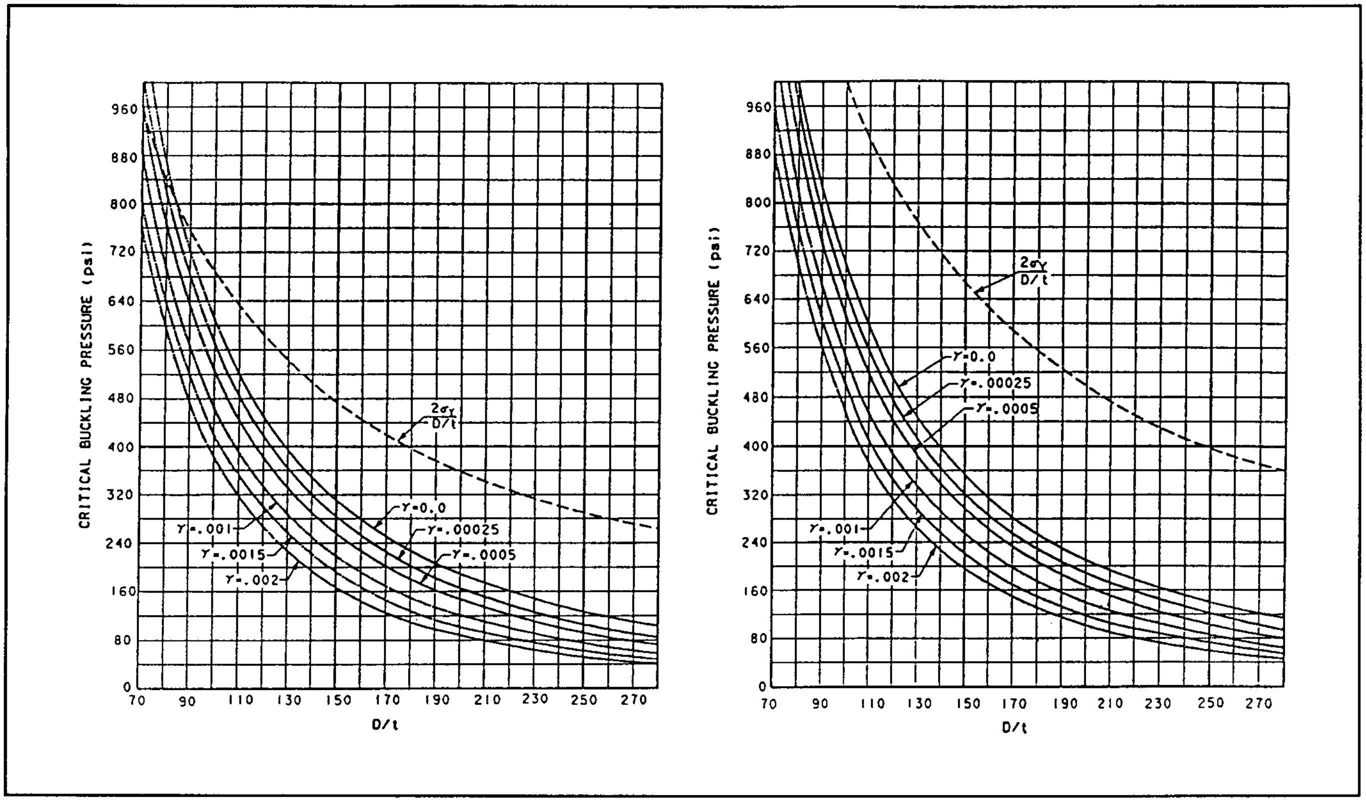


Figure 9-8. Curves based on Amstutz equations by E. T. Moore

E^* = modified modulus of elasticity, $E/(1-\nu_2)$

ν = Poisson's Ratio for steel

Curves based on Jacobsen's equations for the two different steel types are shown on Figure 9-9.

(3) *Vaughan's analysis.* Vaughan's mathematical equation for determination of the critical external buckling pressure is based on work by Bryan and the theory of elastic stability of thin shells by Timoshenko (1936). The failure of the liner due to buckling is not based on the assumption of a single lobe; instead, it is based on distortion of the liner represented by a number of waves as shown in Figure 9-10.

$$\left[\frac{\sigma_y - \sigma_{cr}}{2E^*} + \frac{6\sigma_{cr}}{\sigma_{y} - \sigma_{cr}}\left(\frac{y_o}{R} + \frac{\sigma_{cr}}{E^*}\right)\right] \times \qquad (9\text{-}27)$$

$$\frac{R^2}{T^2} - \frac{R}{T} + \frac{\sigma_y - \sigma_{cr}}{24\sigma_{cr}} = 0$$

where

σ_y = yield stress of liner, psi

σ_{cr} = critical stress

$E^* = E/(1-\nu^2)$

y_o = gap between steel and concrete

R = tunnel liner radius

T = plate thickness

Box 9-6 is a MathCad example of the application of Vaughan's analysis. Vaughan provides a family of curves (Figure 9-11) for estimating approximate critical pressures. These curves are for steel with σ_y = 40,000 psi with various values of y_o/R. It is noted that approximate pressure values obtained from these curves do not include a safety factor.

Box 9-4. MathCad Application of Amstutz's Equations

Liner thickness t = 0.50 in. **ASTM A 516 - 70**

$t := 0.50 \quad F := 0.50 \quad r := 90 \quad k := 0.027 \quad \frac{k}{r} = 3 \cdot 10^{-4}$

$E := 30 \cdot 10^6 \quad \sigma_F := 38 \cdot 10^3 \quad v := 0.30 \quad \frac{t}{2} = 0.25 \quad 2 \cdot \frac{r}{t} = 360$

$\frac{30 \cdot 10^6}{(1 - v^2)} = 3.297 \cdot 10^7 \quad E_m := 3.297 \cdot 10^7 \quad \frac{t}{\sqrt{12}} = 0.144 \quad i := 0.17 \quad \frac{r}{i} = 529.412$

$-\frac{k}{r} \cdot E_m = -9.891 \cdot 10^3 \quad \sigma_v := -9.891 \cdot 10^3$

$$1.5 - 0.5 \cdot \left[\left[\frac{1}{1 + 0.002 \cdot \frac{E}{\sigma_F}}\right]^2\right] = 1.425 \qquad \mu := 1.425$$

$$\frac{\mu \cdot \sigma_F}{\sqrt{1 - v + v^2}} = 6.092 \cdot 10^4 \qquad \sigma^m := 6.092 \cdot 10^4$$

$\sigma_N := 12 \cdot 10^3$

$$a := \sqrt{\left[\left(\frac{\sigma_N - \sigma_v}{\sigma_m - \sigma_N}\right) \cdot \left[\left(\frac{r}{i}\right) \cdot \sqrt{\frac{\sigma_N}{E_m}}\right]^3 - \left[1 - 0.225 \cdot \frac{2 \cdot r}{t} \cdot \left(\frac{\sigma_m - \sigma_N}{E_m}\right)\right] \cdot 1.73 \cdot \frac{2 \cdot r}{t}, \sigma_N\right]}$$

$a = 1.294 \cdot 10^4$

$t := 0.50 \quad F := 0.50 \quad r := 90 \quad \sigma_N := 1.294 \cdot 10^4 \quad i := 0.17 \quad E_m := 3.297 \cdot 10^7 \quad \sigma_m := 6.092 \cdot 10^4$

$$\left(\frac{F}{r}\right) \cdot \sigma_N \cdot \left[1 - 0.175 \cdot \left(\frac{2 \cdot r}{t}\right) \cdot \left(\frac{\sigma_m - \sigma_N}{E_m}\right)\right] = 65.298$$

External pressures:

Critical buckling pressure = 65 psi
Allowable buckling pressure = 43 psi **(Safety Factor = 1.5)**

d. Design examples. There is no one single procedure recommended for analysis of steel liners subjected to external buckling pressures. Available analyses based on various theories produce different results. The results depend, in particular, on basic assumptions used in derivation of the formulas. It is the responsibility of the designer to recognize the limitations of the various design procedures. Use of more than one procedure is recommended to compare and verify final results and to define safe allowable buckling pressures. Most of the steel liner buckling problems can best be solved with MathCad computer applications. Table 9-2 shows the results of MathCad applications in defining allowable buckling pressures for a 90-in. radius (ASTM A 516-70) steel liner with varying plate thicknesses: 1/2, 5/8, 3/4, 7/8, and 1.0 in. Amstutz's and Jacobsen's analyses are based on the assumption of a single-lobe buckling failure. Vaughan's analysis is based on multiple-waves failure that produces much higher

Box 9-5. MathCad Application of Jacobsen's Equations

Liner thickness t = 0.50 in. **ASTM A 516-70**

$t := 0.50 \qquad r := 90 \qquad \Delta := 0.027 \qquad \frac{\Delta}{r} = 3 \cdot 10^{-4}$

$E := 30 \cdot 10^{6} \qquad \sigma_y := 38 \cdot 10^{3} \qquad v := 0.30$

$\frac{30 \cdot 10^{6}}{(1 - v^{2})} = 3.297 \cdot 10^{7} \qquad E_m := 3.296 \cdot 10^{7}$

Guesses $\alpha := 0.35 \qquad \beta := 0.30 \qquad p := 40$

Given

$$\frac{r}{t} = \sqrt{\left[\frac{\left[\left[\frac{9 \cdot \pi^2}{4 \cdot (\beta)^2}\right] - 1\right] \cdot \left[\pi - (\alpha) + (\beta) \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right)^2\right]}{12 \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right)^3 \cdot \left[(\alpha) - \left(\frac{\pi \cdot \Delta}{r}\right) - (\beta) \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right) \cdot \left[1 + \left[\frac{\tan((\alpha) - (\beta))}{4}\right] \cdot \tan((\alpha) - (\beta))\right]\right]}\right]}$$

$$\frac{p}{E_m} = \frac{\left(\frac{9}{4}\right) \cdot \left[\frac{\pi}{(\beta)}\right]^2 - 1}{12 \cdot \left(\frac{r}{t}\right)^3 \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right)^3}$$

$$\frac{\sigma_y}{E_m} = \left(\frac{t}{2 \cdot r}\right) \cdot \left[1 - \left(\frac{\sin(\alpha)}{\sin(\beta)}\right)\right] + \frac{p \cdot r \cdot \sin(\alpha)}{E_m \cdot t \cdot \sin(\beta)} \cdot \left[1 + \frac{4 \cdot (\beta) \cdot r \cdot \sin(\alpha) \cdot \tan((\alpha) - (\beta))}{\pi \cdot t \cdot \sin(\beta)}\right]$$

$$\text{minerr}(\alpha, \beta, p) = \begin{pmatrix} 0.409 \\ 0.37 \\ 51.321 \end{pmatrix}$$

External pressures:

Critical buckling pressure = 51 psi
Allowable buckling pressure = 34 psi **(Safety Factor = 1.5)**

Table 9-2
Allowable Buckling Pressures for a 90-in.-diam. Steel Liner Without Stiffeners

		Plat Thicknesses, in., ASTM A516-70				
Analyses/ Formulas	Safety Factor	1/2	5/8	3/4	7/8	1.0
		Allowable Buckling Pressures, psi				
Amstutz	1.5	65	82	119	160	205
Jacobsen	1.5	51	65	116	153	173
Vaughan	1.5	97	135	175	217	260

allowable buckling pressures. Based on experience, most of the buckling failures involve formation of a single lobe; therefore, use of the Amstutz's and Jacobsen's equations to determine allowable buckling pressures is recommended.

e. Design of steel liners with stiffeners.

(1) *Design considerations.* Use of external circumferential stiffeners should be considered when the thickness of an unstiffened liner designed for external pressure exceeds the thickness of the liner required by the design for internal pressure. Final design should be based on economic considerations of the following three available options that would satisfy the design requirements for the external pressure: (a) increasing the thickness of the liner, (b) adding external stiffeners to the liner using the thickness required for internal pressure, and (c) increasing the

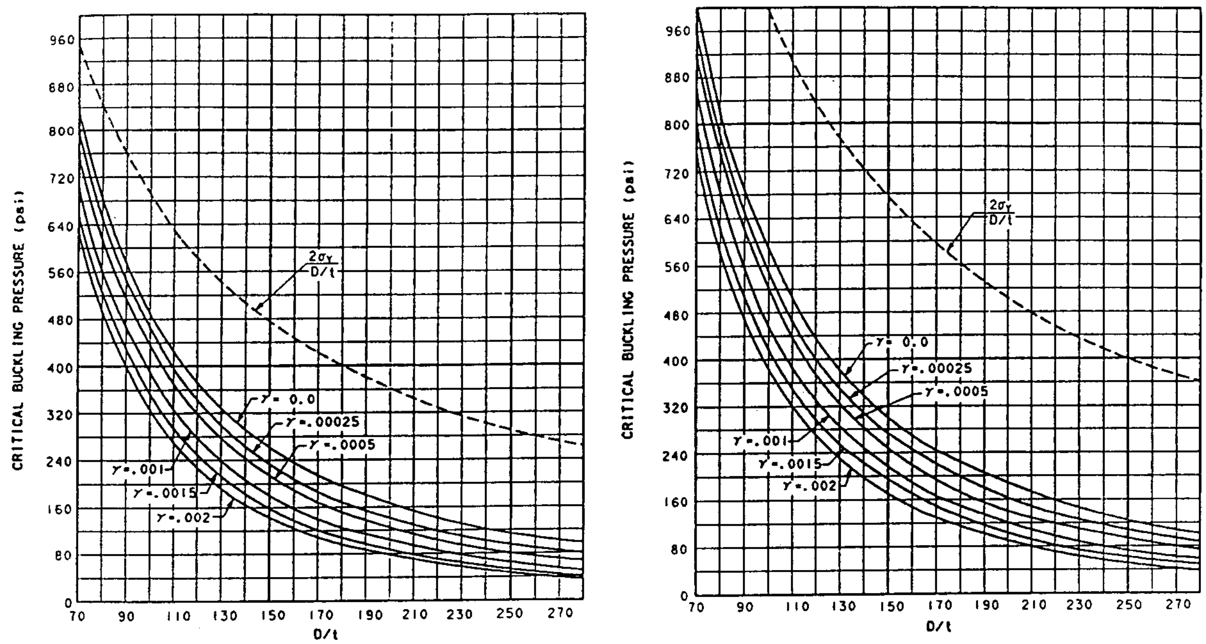


Figure 9-9. Curves based on Jacobsen equations by E. T. Moore

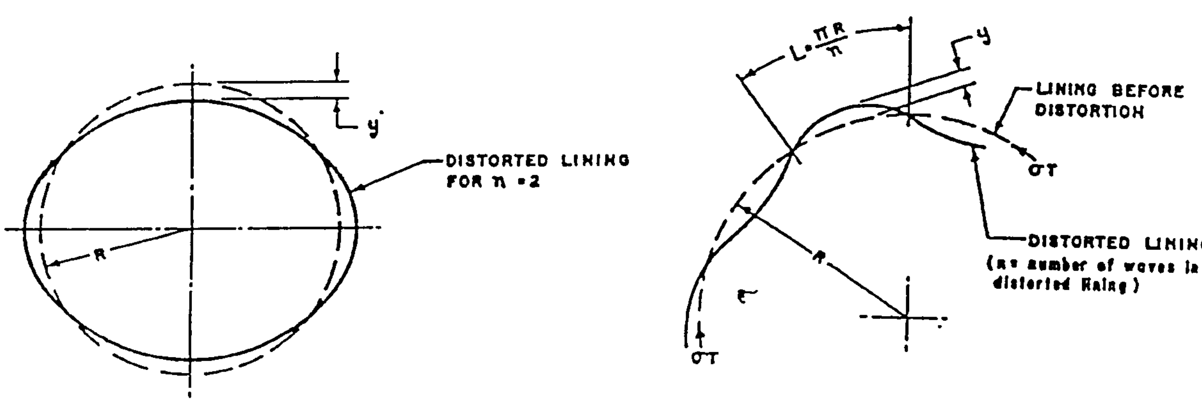


Figure 9-10. Vaughan's buckling patterns - multiple waves

thickness of the liner and adding external stiffeners. The economic comparison between stiffened and unstiffened linings must also consider the considerable cost of additional welding, the cost of additional tunnel excavation required to provide space for the stiffeners, and the additional cost of concrete placement. Several analytical methods are available for design of steel liners with stiffeners. The analyses by von Mises and Donnell are based on distortion of a liner represented by a number of waves, frequently referred to as rotary-symmetric buckling. Analyses by E. Amstutz and by S. Jacobsen are based on a

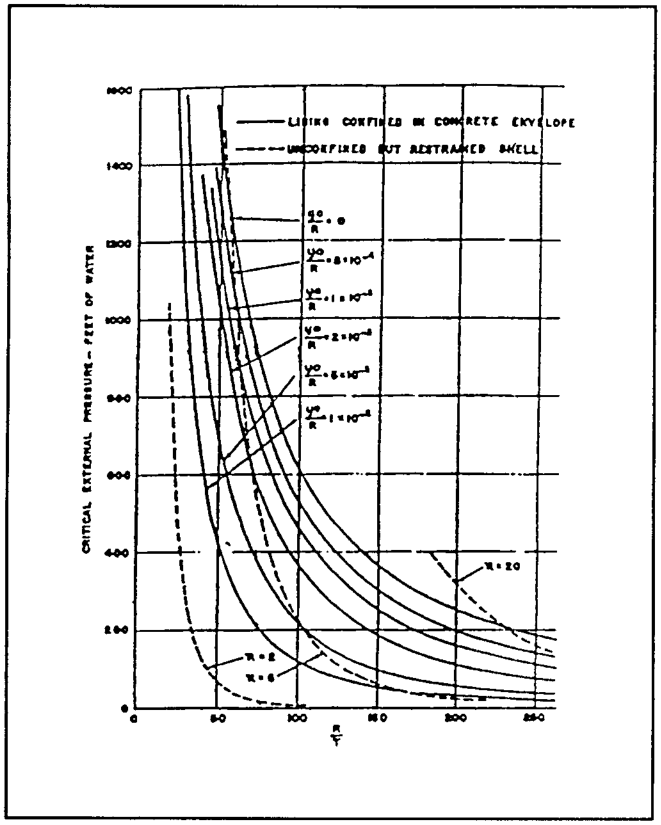


Figure 9-11. Vaughan's curves for yield stress 40,000 psi

single-lobe buckling. Roark's formula is also used. In the single-lobe buckling of liners with stiffeners, the value of ε, an expression for inward deformation of the liner, is generally less than 3; therefore, the corresponding subtended angle 2α is greater than 180° (see Figure 9-7). Since the Amstutz analysis is limited to buckling with ε greater than 3, i.e., 2α less than 180°, it is not applicable to steel liners with stiffeners. For this reason, only Jacobsen's analysis of a single-lobe failure of a stiffened liner is included in this manual, and the Amstutz analysis is not recommended.

(2) *Von Mises's analysis.* Von Mises's equation is based on rotary-symmetric buckling involving formation of a number of waves (n), the approximate number of which can be determined by a formula based on Winderburg and Trilling (1934). A graph for collapse of a free tube derived from von Mises's formula can be helpful in determining buckling of a tube. It is noted that similar equations and graphs for buckling of a free tube have been developed by Timoshenko (1936) and Flügge (1960). Von Mises's equation for determination of critical buckling pressure is:

$$P_{cr} = \frac{E\left(\frac{t}{r}\right)}{1 - \nu^2}\left[\frac{1 - \nu^2}{(n^2 - 1)\left(\frac{n^2 L^2}{\pi^2 r^2} + 1\right)^2}\right] + \frac{E\left(\frac{t}{r}\right)}{12(1 - \nu^2)}\left[n^2 - 1 + \frac{2n^2 - 1 - \nu}{\frac{n^2 L^2}{\pi^2 r^2}} - 1\right] \qquad (9\text{-}28)$$

where

P_{cr} = collapsing pressure psi, for FS = 1.0

r = radius to neutral axis of the liner

ν = Poisson's Ratio

E = modulus of elasticity, psi

t = thickness of the liner, in.

L = distance between the stiffeners, i.e., center-to-center of stiffeners, in.

n = number of waves (lobes) in the complete circumference at collapse

Figure 9-12 shows in graphic form a relationship between critical pressure, the ratio of L/r and the number of waves at the time of the liner collapse. This graph can be used for an approximate estimate of the buckling pressure and the number of waves of a free tube. The number of waves n is an integer number, and it is not an independent variable. It can be determined by trial-and-error substitution starting with an estimated value based on a graph. For practical purposes, $6 \leq n \geq 14$. The number of waves n can also be estimated from the equation by Winderburg and Trilling (1934). The number of waves in the rotary-symmetric buckling equations can also be estimated from the graph shown in Figure 9-12.

(3) *Winderburg's and Trilling's equation.*

Winderburg and Trilling's equation for determination of number of waves n in the complete circumference of the steel liner at collapse is:

$$n = \sqrt[4]{\frac{\frac{3\pi^2}{4\sqrt{(1 - \nu^2)}}}{\left(\frac{L}{D}\right)^2\left(\frac{t}{D}\right)}} \qquad (9\text{-}29)$$

Box 9-6. MathCad Application of Vaughan's Equations

Liner thickness t = 0.50 in. **ASTM A 516-70**

$T := 0.50$ $R := 90$ $\sigma_y := 38 \cdot 10^3$ $y_o := 0.027$ $\frac{y_o}{R} = 3 \cdot 10^{-4}$

$\nu := 0.3$ $\frac{30 \cdot 10^6}{1 - \nu^2} = 3.297 \cdot 10^7$ $E_m := 3.296 \cdot 10^7$ $\sigma_{cr} := 12 \cdot 10^3$

$$a := \sqrt{\left[\left[\frac{\sigma_y - \sigma_{cr}}{2 \cdot E_m} + \frac{6 \cdot \sigma_{cr}}{\sigma_y - \sigma_{cr}} \cdot \left(\frac{y_o}{R} + \frac{\sigma_{cr}}{E_m}\right)\right] \cdot \frac{R^2}{T^2} - \frac{R}{T} + \left(\frac{\sigma_y - \sigma_{cr}}{24 \cdot \sigma_{cr}}\right), \sigma_{cr}\right]}$$

$a = 1.901 \cdot 10^4$ $\sigma_{cr} := 1.901 \cdot 10^4$

$T := 0.50$ $R := 90$ $\sigma_{cr} := 1.901 \cdot 10^4$ $\sigma_m := 6.092 \cdot 10^4$ $E_m := 3.297 \cdot 10^7$

$$\left(\frac{T}{R}\right) \cdot \sigma_{cr} \cdot \left[1 - 0.175 \cdot \left(\frac{2 \cdot R}{T} \cdot \frac{\sigma_m - \sigma_{cr}}{E_m}\right)\right] = 97.153$$

External pressures:

Critical buckling pressure = 97 psi
Allowable buckling pressure = 65 psi **(Safety Factor = 1.5)**

The above equation determines number of waves n for any Poisson's Ratio. For $\nu = 0.3$, however, the above equation reduces to:

$$n = \sqrt[4]{\frac{7.061}{\left(\frac{L}{D}\right)^2 \left(\frac{t}{D}\right)}} \qquad (9\text{-}30)$$

Figure 9-13 shows the relationship between n, length/diameter ratio, and thickness/diameter ratio using this equation.

(4) *Donnell's analysis.*

Donnell's equation for rotary-symmetric buckling is:

$$Pcr = \frac{EI_s}{R^3}\left[\frac{(n^2 + \lambda^2)}{N^2}\right] + \frac{EI_s}{R}\left[\frac{\lambda^2}{n^2\,(n^2 + \lambda^2)^2}\right] \qquad (9\text{-}31)$$

where

P_{cr} = collapsing pressure, for FS = 1.0

R = shell radius, in.

I_s = shell bending stiffness, $t^3/12(1 - \nu^2)$

ν = Poisson's Ratio

E = modulus of elasticity

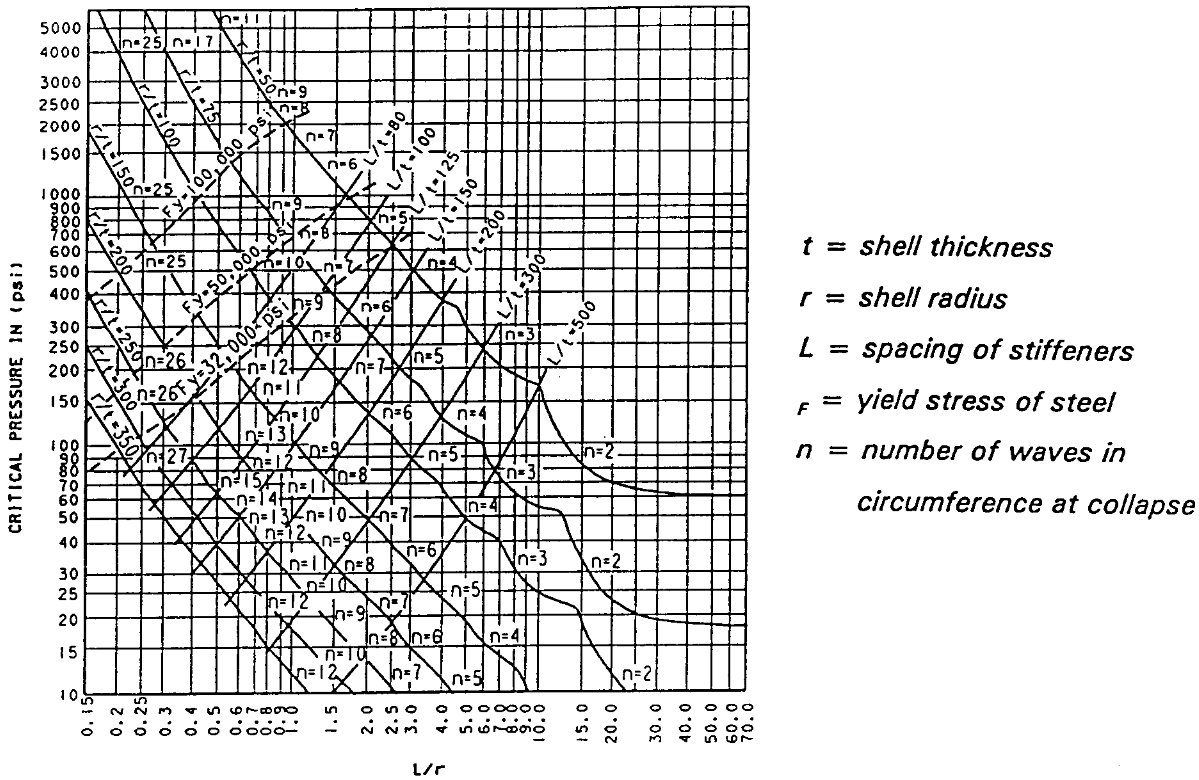


Figure 9-12. Collapse of a free tube (R. von Mises)

t = shell thickness

$\lambda = \pi R/L$

L = length of tube between the stiffeners

n = number of waves (lobes) in the complete circumference at collapse

(5) *Roark's formula.* When compared with other analyses, Roark's formula produces lower, safer, critical buckling pressures. Roark's formula for critical buckling is:

$$P_{cr} = \frac{0.807\ E_s\ t^2}{L_1\ R_1}\ \sqrt[4]{\left(\frac{1}{1 - v^2}\right)^3 \frac{t^2}{R_1^2}} \qquad (9\text{-}32)$$

where

E = modulus of elasticity of steel

t = thickness of the liner

R_1 = radius to the inside of the liner

v = Poisson's ratio for steel

L_1 = spacing of anchors (stiffeners)

(6) *Jacobsen's equations.* Jacobsen's analysis of steel liners with external stiffeners is similar to that without stiffeners, except that the stiffeners are included in computing the total moment of inertia, i.e., moment of inertia of the stiffener with contributing width of the shell equal to $1.57\sqrt{rt} + t_s$. As in the case of unstiffened liners, the analysis of liners with stiffeners is based on the assumption of a single-lobe failure. The three simultaneous equations with three unknowns α, β, and *p* are:

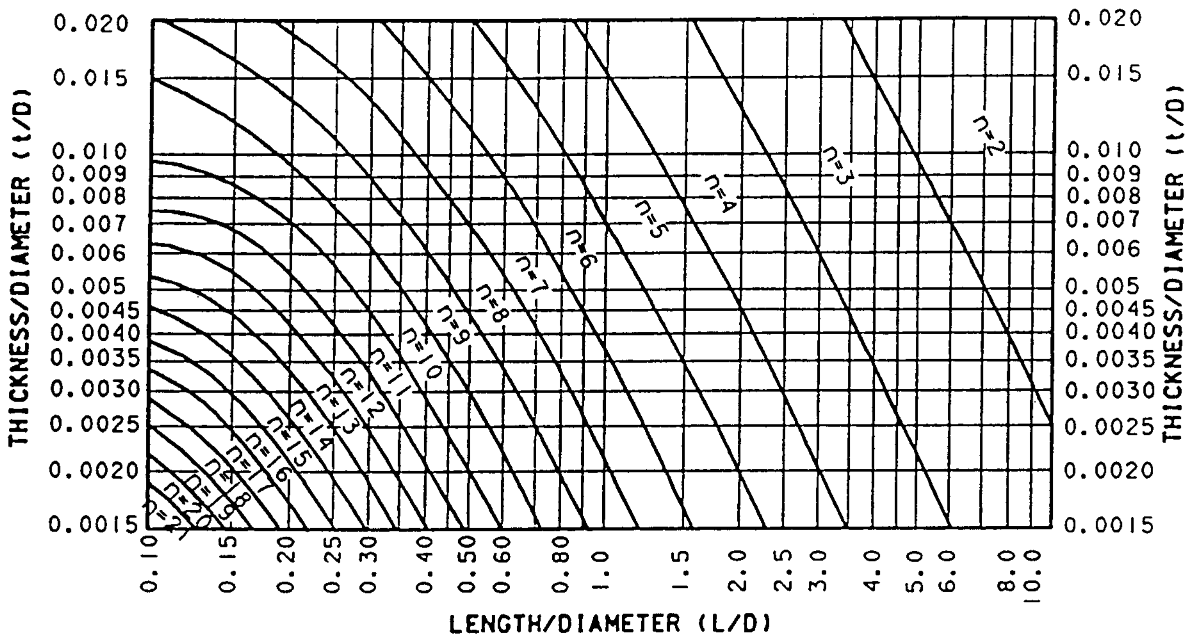


Figure 9-13. Estimation of *n* (Winderburg and Trilling)

$$r/\sqrt{(12J/F)} = \left\{ \frac{[(9\pi^2/4\beta^2) - 1]\ [\pi - \alpha + \beta(\sin\alpha/\sin\beta)^2]}{12(\sin\alpha/\sin\beta)^3\ [\alpha - (\pi\Delta/r) - \beta(\sin\alpha/\sin\beta)\ [1 + \tan^2(\alpha - \beta)/4]]} \right\}^{1/2} \tag{9-33}$$

$$(p/EF) = \frac{[(9\pi^2/4\beta^2) - 1]}{(r^3 \sin^3\alpha)/[(J/F)\sqrt{\sin^3\beta}\,]} \tag{9-34}$$

$$\sigma_y/I = \frac{h}{r}\left(1 - \frac{\sin\beta}{\sin\alpha}\right) + \frac{p\ r\ \sin\alpha}{EF\ \sin\beta}\left[1 + \frac{8\ \alpha\ hr\ \sin\alpha\ \tan(\alpha - \beta)}{\pi\ \sin\beta\ 12J/F}\right] \tag{9-35}$$

where

α = one-half the angle subtended to the center of the cylindrical shell by the buckled lobe

β = one-half the angle subtended by the new mean radius through the half waves of the buckled lobe

p = critical external buckling pressure

J = moment of inertia of the stiffener and contributing width of the shell

F = cross-sectional area of the stiffener and the pipe shell between the stiffeners

h = distance from neutral axis of stiffener to the outer edge of the stiffener

r = radius to neutral axis of the stiffener

σ = yield stress of the liner/stiffener

E = modulus of elasticity of liner/stiffener

Δ/r = gap ratio, i.e., gap/liner radius

Box 9-7 shows a MathCad application of Jacobsen's equation.

(7) *Examples.* Von Mises's and Donnell's equations for rotary-symmetric buckling can best be solved by MathCad application. MathCad application does not require a prior estimate of number of waves n in the circumference of the steel liner at collapse. Instead, a range of n values is defined at the beginning of either equation and, as a result, MathCad produces a range of values for critical pressures corresponding to the assumed n values. Critical pressures versus number of waves are plotted in graphic form. The lowest buckling pressure for each equation is readily determined from the table produced by

Box 9-7. Liner with Stiffeners-Jacobsen Equations

Liner thickness t = 0.500 in.
Stiffeners: 7/8" x 6" @ 48 in. on centers

$r := 90$ $\quad J := 44.62$ $\quad F := 29.25$ $\quad E := 30 \cdot 10^6$

$\Delta := 0.027$ $\quad \frac{\Delta}{r} = 3 \cdot 10^{-4}$ $\quad h := 4.69$ $\quad \sigma_y := 38 \cdot 10^3$

Guesses $\quad \alpha := 1.8$ $\quad \beta := 1.8$ $\quad p := 125$

Given

$$\frac{r}{\sqrt{\left[\frac{12 \cdot J}{F}\right]}} = \sqrt{\left[\frac{\left[\left[\frac{9 \cdot \pi^2}{4 \cdot (\beta)^2}\right] - 1\right] \cdot \left[\pi - (\alpha) + (\beta) \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right)^2\right]}{12 \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right)^3 \cdot \left[(\alpha) - \left(\frac{\pi \cdot \Delta}{r}\right) - (\beta) \cdot \left(\frac{\sin(\alpha)}{\sin(\beta)}\right) \cdot \left[1 + \frac{\tan((\alpha) - (\beta))^2}{4}\right]\right]}\right]}$$

$$\left(\frac{p}{E \cdot F}\right) \cdot \sqrt{\left(\frac{12 \cdot J}{F}\right)} = \frac{\left[\left[\frac{9 \cdot \pi^2}{4 \cdot (\beta)^2}\right] - 1\right]}{\left[\frac{r^3 \cdot \sin(\alpha)^3}{\frac{J}{F} \cdot \sqrt{\frac{12 \cdot J}{F}} \cdot \sin(\beta)^3}\right]}$$

$$\frac{\sigma_y}{E} = \frac{h \cdot \sqrt{\frac{12 \cdot J}{F}}}{R \cdot \sqrt{\left(\frac{12 \cdot J}{F}\right)}} \cdot \left(1 - \frac{\sin(\beta)}{\sin(\alpha)}\right) + \frac{p \cdot \sqrt{\frac{12 \cdot J}{F}} \cdot r \cdot \sin(\alpha)}{E \cdot F \cdot \sqrt{\left(\frac{12 \cdot J}{F}\right)} \cdot \sin(\beta)} \cdot \left[1 + \frac{8 \cdot (\beta) \cdot h \cdot r \cdot \sin(\alpha) \cdot \tan((\alpha) - (\beta))}{\pi \cdot \sqrt{\frac{12 \cdot J}{F}} \cdot \sqrt{\left(\frac{12 \cdot J}{F}\right)} \cdot \sin(\beta)}\right]$$

$p < 130$

$$minerr(\alpha, \beta, p) = \begin{pmatrix} 1.8 \\ 1.8 \\ 126.027 \end{pmatrix}$$

External pressures:

Pcr. (critical buckling pressure) = 126 psi
Pall. (allowable buckling pressure) = 84 psi

MathCad computations. Design examples for determination of critical buckling pressures are included in Boxes 9-8, 9-9, and 9-10. Number of waves in the complete circumference at the collapse of the liner can best be determined with MathCad computer applications as shown in Box 9-11. Table 9-3 below shows that allowable buckling pressures differ depending on the analyses used for computations of such pressures. A designer must be cognizant of such differences as well as the design limitations of various procedures to determine safe allowable buckling pressures for a specific design. An adequate safety factor must be used to obtain safe allowable pressures, depending on a specific analysis and the mode of buckling failure assumed in the analysis.

f. Transitions between steel and concrete lining. In partially steel-lined tunnels, the transition between the steel-lined and the concrete-lined portions of the tunnel requires special design features. Seepage rings are usually installed at or near the upstream end of the steel liner. One or more seepage rings may be required. ASCE (1993) recommends three rings for water pressures above 240 m (800 ft) (see Figure 9-14). A thin liner shell may be provided at the transition, as shown on Figure 9-14 with studs, hooked bars, U-bars, or spirals installed to prevent buckling. Alternatively, ring reinforcement designed for crack control may be provided for a length of about twice the tunnel diameter, reaching at least 900 mm (3 ft) in behind the steel lining. Depending on the character of the rock and the method of construction, a grout curtain may be provided to minimize water flow from the concrete-lined to the steel-lined section through the rock.

g. Bifurcations and other connections. Bifurcations, manifolds, and other connections are generally designed in accordance with the principles of aboveground penstocks, ignoring the presence of concrete surrounding the steel structure. The concrete may be assumed to transfer unbalanced thrust forces to competent rock but is not assumed otherwise to help support internal pressures. Guidance in the design of these structures is found in EM 1110-2-2902, Conduits, Culverts and Pipes, and EM 1110-2-3001. Steel lining connections are usually straight symmetrical or asymmetrical wyes. Right-angle connections should be avoided, as they have higher hydraulic resistance. These connections require reinforcement to replace the tension resistance of the full-circle steel circumference interrupted by the cut in the pipe provided for the connection. The reinforcement can take several forms depending on the pressure in the pipe, the pipe size, and the pipe connection geometry. This is expressed by the pressure-diameter value (PDV), defined as

$$PDV = pd^2/(D \sin^2 \alpha) \tag{9-38}$$

where

p = design pressure, psi

d = branch diameter, in.

D = main diameter, in.

α = branch deflection angle

Depending on the PDV, the reinforcement should be applied as a collar, a wrapper, or a crotch plate. Collars and wrappers are used for smaller pipes where most tunnels would employ crotch plates. These usually take the shape of external plates welded onto the connection between the pipes. The selection of steel reinforcement is made according to Table 9-4. The external steel plate design depends on the geometry and relative pipe sizes. One or more plates may be used, as shown in the examples on Figure 9-15. Because space is limited around the steel lining in a tunnel, it is often practical to replace the steel reinforcement plate with an equivalent concrete reinforcement. For a collar or wrapper, the reinforcement plate should be equal in area to the steel area removed for the connection, except that for PDV between 4,000 lb/in. and 6,000 lb/in., this area should be multiplied by PDV times 0.00025.

Box 9-8. Liner with Stiffeners - Roark's Formula

Liner Thickness t = 1/2, 5/8, 3/4, 7/8, and 1.00 in.
Siffeners: 7/8" x 6" and larger for thicker liners @ 48 in. on centers

Design data:

R_1 = 90 in. - radius to the inside of the liner
t = 1/5, 5/8, 3/4, 7/8, and 1.00 in. - selected range of liner thicknesses
E_s = 30,000,000 psi - modulus of elasticity
υ = 0.3 - Poisson's Ratio
L_1 = 48 in. - spacing of stiffeners
Pcr = "d(t)" - critical (collapsing) pressure for factor of safety F.S. = 1.0

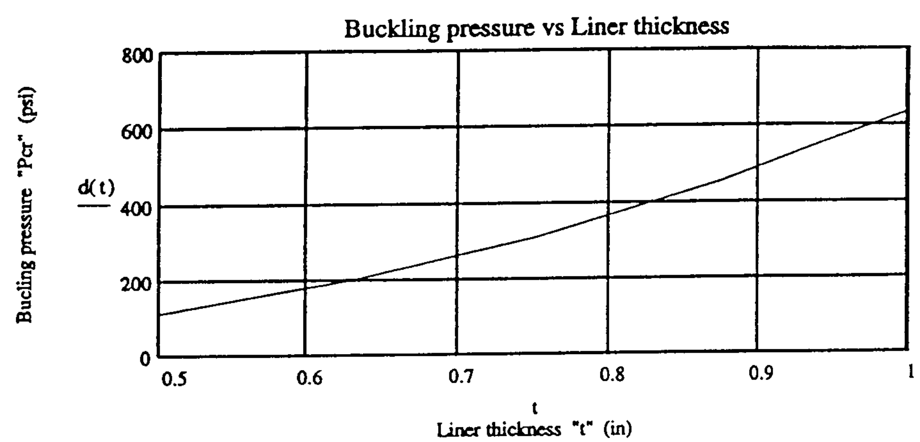
t: = 0.50, 0.625.. 1.00

$R_1 := 90$ $\quad L_1 := 48$ $\quad \upsilon_s := 0.3$ $\quad E_s := 30 \cdot 10^6$

$$d(t) := \frac{0.807 \cdot E_s \cdot t^2}{L_1 \cdot R_1} \cdot \left[\left(\frac{1}{1 - \nu_s^2}\right)^3 \cdot \frac{t^2}{R_1^2}\right]^{0.25}$$

- critical buckling pressure formula

d(t)
112.081
195.798
308.86
454.076
634.028



External pressures:

t (thickness), in.	Pcr, psi	Pall, psi F.S. = 1.5	Pall, psi F.S. = 2.0
1/2	112	75	66
5/8	196	131	98
3/4	309	206	154
7/8	454	303	227
1.0	634	423	317

Box 9-9. Liner with Stiffeners - R. von Mises's Equation

Liner Thickness t = 0.50 in.
Stiffeners: 7/8" x 6" @ 48 in. on centers

Design data:

r = 90 in. - radius to neutral axis of shell (for practical purposes, radius to outside of shell)
L = 48 in. - length of liner between stiffeners, i.e., center-to-center spacing of stiffeners
t = 0.50 in. - thickness of the liner
E = 30,000,000 psi - modulus of elasticity
υ = 0.3 - Poisson's Ratio
n = number of lobes or waves in the complete circumference at collapse
Pcr = d(n) - critical (collapsing) pressure for factor of safety F.S. = 1.0

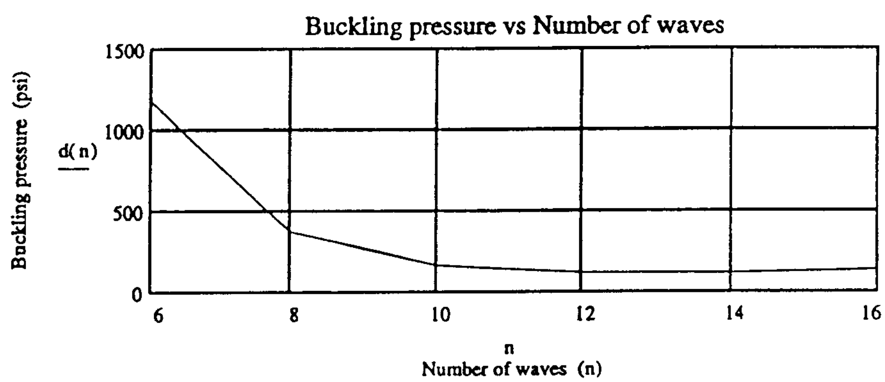
$n := 6, 8 .. 16$

$t := 0.50 \qquad r := 90 \qquad L := 48 \qquad \upsilon := 0.3 \qquad E := 30 \cdot 10^{6}$

$$d(n) := \frac{E \cdot \left(\frac{t}{r}\right)}{1 - \nu^2} \cdot \left[\frac{1 - \nu^2}{(n^2 - 1)} \cdot \left(\frac{n^2 \cdot L^2}{\pi^2 \cdot r^2} + 1\right)^{2}\right] + \frac{E \cdot \left(\frac{t}{r}\right)^3}{12 \cdot (1 - \nu^2)} \cdot \left[n^2 - 1 + \frac{2 \cdot n^2 - 1 - \nu}{\left(\frac{n^2 \cdot L^2}{\pi^2 \cdot L^2}\right)} - 1\right]$$

- critical buckling pressure equation

d(n)
$1.76 \cdot 10^3$
367.522
168.596
121.242
120.951
139.08



External pressures:

Pcr (critical buckling pressure) = 120 psi

Pall (allowable buckling pressure) = 80 psi

Box 9-10. Liner with Stiffeners-Donnell's Equation

Liner Thickness t = 0.50 in.
Stiffeners: 7/8" x 6" @ 48 in. on centers

Design data:

R = 90 in. - shell radius
L = 48 in. - length of liner between stiffeners, i.e., center-to-center spacing of stiffeners
t = 0.50 in. - thickness of the liner
E = 30,000,000 psi - modulus of elasticity
υ = 0.3 - Poisson's Ratio
n = number of lobes or waves in the complete circumference at collapse
Pcr = d(n) - critical (collapsing) pressure for factor of safety F.S. = 1

$n := 6, 8 .. 16$

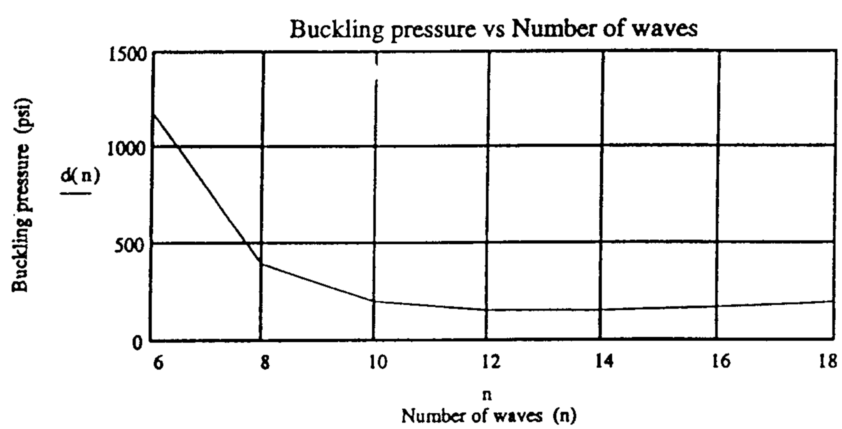
$t := 0.50$ $\quad R := 90$ $\quad L := 48$ $\quad \upsilon := 0.3$ $\quad \lambda := \frac{\pi \cdot R}{L}$ $\quad I_s := \frac{t^3}{12 \cdot (1 - \upsilon^2)}$

$\lambda = 5.89$ $\quad I_s = 0.011$ $\quad E := 30 \cdot 10^6$

$$d(n) := \frac{E \cdot I_s}{R^3} \cdot \left[\frac{(n^2 + \lambda^2)^2}{n^2}\right] + \frac{E \cdot t}{R} \cdot \left[\frac{\lambda^4}{n^2 \cdot (n^2 + \pi^2)^2}\right]$$

-- critical buckling pressure equation

d(n)
$1.181 \cdot 10^3$
393.553
196.062
148.098
147.148
164.773
191.879



External pressures:

Pcr (critical buckling pressure) = 147 psi

Pall (allowable buckling pressure) = 98 psi (with safety factor F.S. = 1.5)

Box 9-11. Determination of Number of Waves (lobes) at the Liner Collapse

Liner Thicknesses : t = 1/2, 5/8/, 3/4, 7/8 and 1.0 in.
Stiffener spacing @ 48 in. on centers

Design data:

D = 180 in. - tunnel liner diameter
L = 48 in. - spacing of stiffeners
υ = 0.3 - Poisson's Ratio
t = 1/2, 5/8, 3/4, 7/8 and 1.0 in. - selected range of liner thicknesses
n = "d(t)" - number of waves (lobes) in the complete circumference at collapse

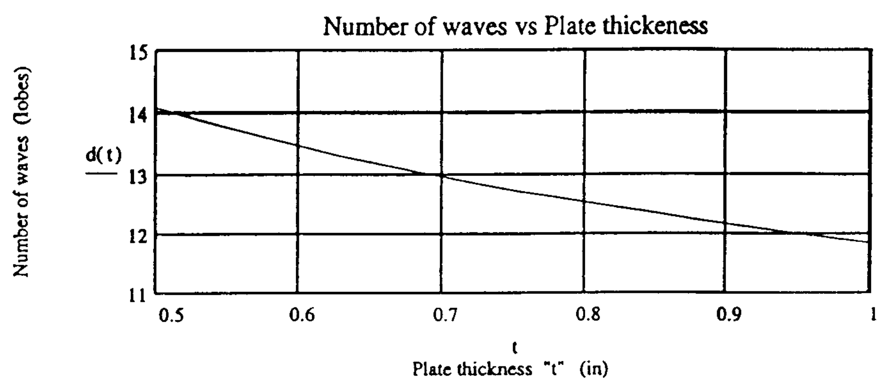
t : = 0.50, 0.625 .. 1.00

D : = 180 L : = 48 υ : = 0.3

$$d(t) := \left[\frac{\dfrac{3\cdot\pi^2}{4\cdot\sqrt{1-\nu^2}}}{\left(\dfrac{L}{D}\right)^2\cdot\left(\dfrac{t}{D}\right)}\right]^{0.25}$$

- - Winderburg and Trilling formula for υ = 0.3

d(t)
14.078
13.314
12.721
12.24
11.838



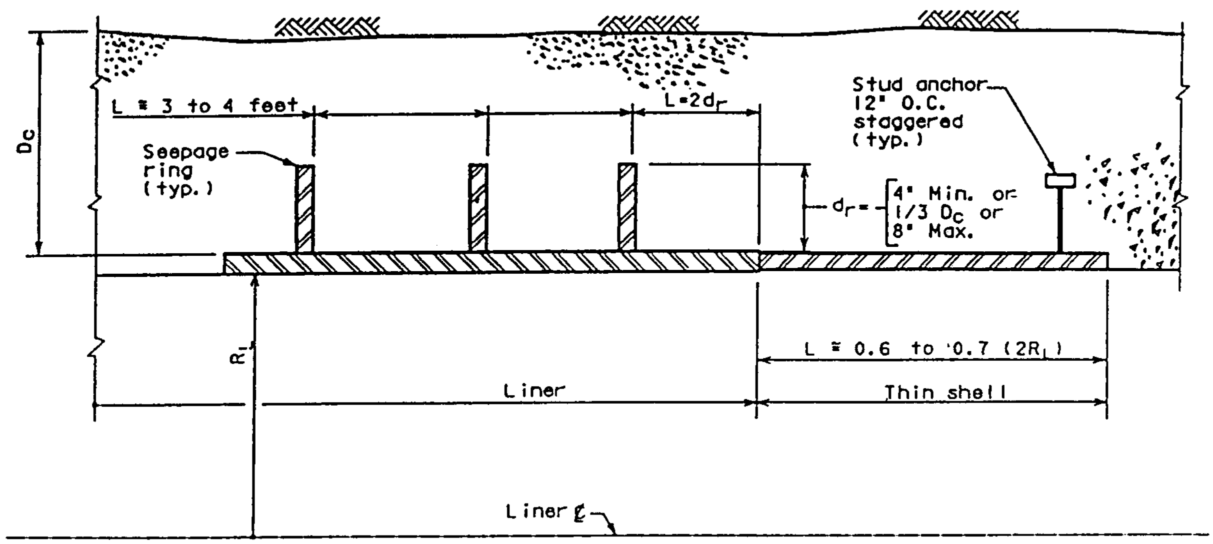


Figure 9-14. Seepage ring and thin shell configuration

Table 9-3
Allowable Buckling Pressures for a 90-in.-diam. Steel Liner With Stiffeners Spaced 48 in.

		Plate Thicknesses, in. (ASTM A516-70)				
Analyses/ Formulas	Safety Factor	1/2	5/8	3/4	7/8	1.0
		Allowable Buckling Pressures, psi				
Roark	1.5	75	131	206	303	423
Von Mises	1.5	80	137	218	327	471
Donnell	1.5	98	172	279	424	603
Jacobsen	1.5	84	143	228	348	482

Table 9-4

PDV (lb/in.)		>6,000	4,000-6,000	<4,000
d/D	>0.7		Crotch wrapper plate	Wrapper
	<0.7		Crotch collar or plate wrapper	Collar or wrapper

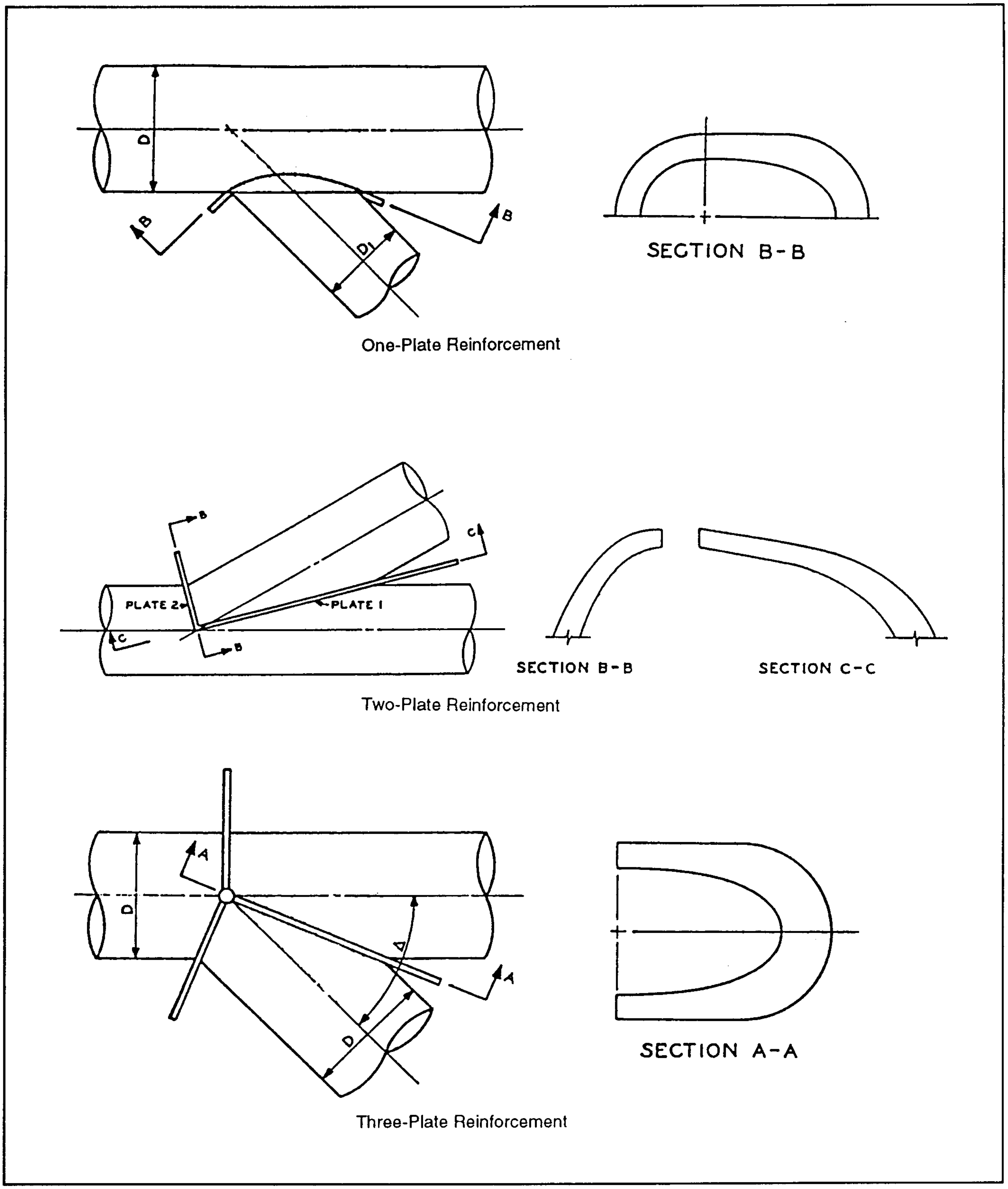


Figure 9-15. Steel-lining reinforcement